BRL-TR-3097

DTIC FILE COPY

(2)

TECHNICAL REPORT BRL-TR-3097

BRL

AD-A222 590

NEW PRESSURE GRADIENT EQUATIONS FOR LUMPED-PARAMETER INTERIOR BALLISTIC CODES

FREDERICK W. ROBBINS
RONALD D. ANDERSON
PAUL S GOUGH

MAY 1990

U.S. ARMY LABORATORY COMMAND

BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

90 06 11 059

NOTICES

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (*Leave blank*)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	May 1990	Final Jan 86-Jan 90

4. TITLE AND SUBTITLE

New Pressure Gradient Equations for
Lumped-Parameter Interior Ballistic Codes

5. FUNDING NUMBERS

PR: 1L161102AH43

6. AUTHOR(S)

Frederick W. Robbins, Ronald D. Anderson and Paul S. Gough

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

USA Ballistic Research Laboratory
ATTN: SLCBR-DD-T
Aberdeen Proving Ground, MD 21005-5066

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

BRL-TR-3097

11. SUPPLEMENTARY NOTES

The authors wish to thank T.S. Raab and K.A. Cieslewicz for performing the many required computer runs and the preparation of many plots. We also acknowledge the assistance of G. Wren and Dr. W. Morrison of the Ballistic Research Laboratory, and Dr. R. Heiser of Fraunhofer-Institut für Kurzzeitdynamik Ernst-Mach-Institut, Weil am Rhein, West Germany, for their excellent reviews.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (*Maximum 200 words*)

Lumped-parameter models of interior ballistics require a relation between the space mean pressure and the pressure at the base of the projectile. Typically, this is furnished using the closed form analyses of Lagrange or of Pidduck and Kent. These relations do not take into account the non-uniform cross-sectional area of the chamber of the gun and the two-phase structure of the flow propellant grains and their products of combustion.

THIS ~~In the present~~ report ~~we~~ describes analytical relations which reflect first the variation in cross-sectional area, second the two-phase structure of the flow and third, the combination of both variable area and two-phase flow. These three sets of relations are programmed into a lumped-parameter code the results of which are compared with those from a quasi-one-dimensional two-phase flow code. Significant improvements in predictive capacity are observed relative to the traditional Lagrange formula when high performance charges are considered. Keywords:

14. SUBJECT TERMS

Gradients, Gradient Equations, Interior Ballistics, Lumped-Parameter, Chambrage, Two-Phase Flow, RGA, IBRGA

15. NUMBER OF PAGES

156

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

NSN 7540-01-280-5500 (KR)

UNCLASSIFIED

Standard Form 298 (Rev 2-89)
Prescribed by ANSI Std Z39-18
298-102

INTENTIONALLY LEFT BLANK.

TABLE OF CONTENTS

INTENTIONALLY LEFT BLANK.

LIST OF FIGURES

INTENTIONALLY LEFT BLANK.

LIST OF TABLES

INTENTIONALLY LEFT BLANK.

ACKNOWLEDGMENTS

The authors wish to thank T.S. Raab and K.A. Cieslewicz for performing the many required computer runs and the preparation of many plots. We also acknowledge the assistance of G. Wren and Dr. W. Morrison of the Ballistic Research Laboratory, and Dr. R. Heiser of Fraunhofer-Institut für Kurzzeitdynamik Ernst-Mach-Institut, Weil am Rhein, West Germany, for their excellent reviews.

INTENTIONALLY LEFT BLANK.

1. INTRODUCTION

During the analysis of 120-mm gun firings, designed to look at the interior ballistic characteristics of combustible cartridge cases (Robbins, Koszoru, and Minor 1986), XNOVAKTC (XKTC) (Gough 1980) was noted to be in agreement with measured pressure-time curves as well as with pressure-difference curves, while IBHVG2 (Anderson and Fickie, 1987) gave a calculated maximum breech pressure which was 42 MPa higher than measured. Parametric studies were performed using XKTC to attempt to attribute this disparity to the various processes omitted from IBHVG2. The boattail instrusion was calculated to account for 14 MPa, with effects of flamespreading and intergranular stress accounting for 3 MPa each. Subsequent calculations (Robbins 1986) indicated chambrage, propellant packaging, wave dynamics, and multiphase effects (the solid propellant velocity lag and concomitant formation of an ullage region between the projectile base and the propellant bed) as contributors to the differences between the lumped-parameter and two-phase interior ballistic codes. In this report, we show that the influence of chambrage and propellant velocity lag on the pressure gradient may be represented in analytical form. We compare the analytical pressure gradient with that predicted by XKTC and assess the extent to which the effects of chambrage and propellant velocity lag account for the differences in ballistic predictions.

The NOVA codes, of which XKTC is the latest version, have been used with uncompromised data bases to model gun systems with much success (Robbins, Koszoru, and Minor 1986; Robbins 1983; Robbins and Horst 1984). Since XKTC calculates the pressure gradient from first principles and agrees with gun firings, XKTC is assumed correct. Accordingly, all the lumped-parameter computer runs, with different gradient equations, are compared with equivalent XKTC computer runs.

1.1 Models. Gradient equations have been developed by Gough (Gough, no date) to look at (1) the effects of chambrage, (2) the effects of propellant velocity lag, and (3) both effects at the same time. The governing partial differential equations, assumptions, and definitions are given for each effect followed by the resultant gradient equations and an equation for the total kinetic energy of the propellant and gas. The details of the solution technique and mathematical procedure are documented in Appendices 1-3.

1.2 Influence of Chambrage. For the chambrage gradient equation, the propellant is assumed to be uniformly distributed between the breech and the base of the projectile, and the variation in area is assumed to be confined to the chamber. The basic results presented in this section have been previously derived (Vinti 1942) and, more recently, in a somewhat different context, (Morrison and Wren 1988).

The continuity and momentum equations for unsteady flow of a homogeneous inviscid substance are

$$\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial \rho A u}{\partial z} = 0 \qquad (C.01)$$

and

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial z} + g_o \frac{\partial p}{\partial z} = 0. \qquad (C.02)$$

Making the Lagrange assumption

$$\frac{\partial \rho}{\partial z} = 0 \qquad (C.03)$$

and using

$$\rho = \frac{C}{V(z_p)} \qquad (C.04)$$

results in the pressure distribution

$$p(z) = p(0) + a(t)J_1(z) + b(t)J_2(z), \qquad (C.05)$$

where

$$a(t) = \frac{CA_B}{g_o V^2(z_p)} \left[\frac{A_B V_p^2}{V(z_p)} - \dot{V}_p \right], \qquad (C.06)$$

$$b(t) = -\frac{CV_p^2 A_B^2}{2g_o V^3(z_p)}, \qquad (C.07)$$

$$J_1(z) = \int_o^z \frac{V(z)dz}{A(z)}, \qquad (C.08)$$

and

$$J_2(z) = \frac{V^2(z)}{A^2(z)}. \qquad (C.09)$$

Substituting the projectile acceleration $\dot{V}_p$ given by

$$\dot{V}_p = \frac{g_o A_B}{M_p} [P_B - P_{res}] \qquad (C.10)$$

into (C.05) results in a pressure distribution in terms of the pressure at the base of the projectile P_B

$$p(z) = p(0) + a_1(t)J_1(z) + a_2(t)J_1(z)P_B + b(t)J_2(z), \qquad (C.11)$$

where

$$a_1(t) = \frac{CA_b}{g_o V^2(z_p)} \left[\frac{A_B V_p^2}{V(z_p)} + \frac{g_o A_B}{M_p} P_{res} \right] \qquad (C.12)$$

and

$$a_2(t) = -\frac{CA_b^2}{M_p V^2(z_p)}. \qquad (C.13)$$

With the mean pressure P_m defined as

$$P_m = \frac{\int_o^{z_p} P(z)A(z)dz}{\int_o^{z_p} A(z)dz} \qquad (C.14)$$

and using (C.11) and identifying P(0) with P_{Br}, then the base and breech pressures are given in terms of the mean pressure by

$$P_B = \frac{P_m + a_1(t)J_1(z_p) + b(t)J_2(z_p) - \dfrac{a_1(t)J_3(z_p)}{V(z_p)} - \dfrac{b(t)J_4(z_p)}{V(z_p)}}{1 - a_2(t)J_1(z_p) + \dfrac{a_2(t)J_3(z_p)}{V(z_p)}} \qquad (C.15)$$

and

$$P_{Br} = (1 - a_2(t)J_1(z_p))\ P_B - a_1(t)J_1(z_p) - b(t)J_2(z_p), \qquad (C.16)$$

where

$$J_3(z_p) = \int_o^{z_p} A(z)J_1(z)dz \qquad (C.17)$$

and

$$J_4(z_p) = \int_o^{z_p} A(z)J_2(z)dz. \qquad (C.18)$$

The kinetic energy of the gas/solid mixture is

$$KE = \frac{A_B^2 V_p^2 C J_4(z_p)}{2g_o V^3(z_p)}. \qquad (C.19)$$

1.3 Influence of Propellant Velocity Lag. For the two-phase gradient equation developed in this section, the chamber and tube are assumed to have the same, uniform diameter. The coupling of the two-phase effects with chambrage is addressed in the next section. The propellant is assumed to be uniformly distributed between the breech face and the leading edge of the bed with the leading edge of the bed being initially at the base of the projectile. However, once the projectile begins to move, the leading edge of the propellant bed is not assumed to remain in contact with the base of the projectile. Instead, the motion of the leading edge of the bed is explicitly modeled by reference to the equation of motion of the solid phase which includes the influence of both pressure gradient (buoyancy force) and interphase drag. The existence of a region of

ullage between the propellant bed and the base of the projectile is also recognized explicitly. The requirement that the gas-phase density be everywhere uniform causes the gas velocity at the leading edge of the propellant bed (U_g) to have a much more complex time dependence than that in the simple Lagrange analysis. Instead of being a fixed fraction of the projectile velocity at all times, the value of U_g may be significantly greater than the projectile velocity during the pressurization phase since the ullage requires a net compression to maintain equilibrium with the conditions in the mixture region where combustion is occurring. The complex time dependence of U_g will be seen to have important repercussions in respect to the behavior of the pressure gradient. It is believed that this is the first time that these effects have been modeled analytically.

The continuity equations for the mixture region are

$$\frac{\partial \varepsilon\rho}{\partial t} + \frac{\partial \varepsilon\rho u}{\partial z} = \dot{m}(t) \qquad (L.1)$$

$$\rho_p \frac{\partial \varepsilon}{\partial t} - \frac{\partial[(1-\varepsilon)\rho_p u_p]}{\partial z} = \dot{m}(t). \qquad (L.2)$$

Making assumptions analogous to the Lagrange assumption

$$\frac{\partial \rho}{\partial z} = 0 \quad (\textit{gas density constant throughout the tube}) \qquad (L.3)$$

$$\frac{\partial \varepsilon}{\partial z} = 0 \quad (\textit{porosity constant throughout the mixture region}) \qquad (L.4)$$

and noting the assumption $\dot{m}(t)$ is only a function of time, then

$$u_p = U_p \left[\frac{z}{z_b}\right] \qquad (L.5)$$

and

$$u = U_g \left[\frac{z}{z_b}\right]. \qquad (L.6)$$

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form

$$\varepsilon \ (U_g - U_p) = U_{g+} - U_p.$$

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption L.3.

Consider the momentum equation for the mixture region,

$$\frac{\partial \varepsilon \rho u}{\partial t} + \frac{\partial \varepsilon \rho u^2}{\partial t} + \varepsilon g_o \frac{\partial P}{\partial z} = - f_s + \dot{m} u_p \tag{L.7}$$

and

$$\frac{\partial (1-\varepsilon)\rho_p u_p}{\partial t} + \frac{\partial (1-\varepsilon)\rho_p u_p^2}{\partial z} + (1-\varepsilon) g_o \frac{\partial P}{\partial z} = f_s - \dot{m} u_p, \tag{L.8}$$

where

$$f_s = \frac{(1-\varepsilon)}{D_p} \rho (u-u_p)^2 f_{sc}, \tag{L.9}$$

and for

$$REN = \frac{\rho D_p | U_g - U_p |}{\mu}$$

$$\lambda = \left[\frac{0.5 + \frac{GL}{GD}}{\left(1.5 \frac{GL}{GD}\right)^{2/3}} \right]^{2.17}$$

$$\varepsilon_o = 1 - \frac{C}{\rho_p V(z_{p_o})}.$$

Then

$$f_{sc} = \begin{cases} \dfrac{2.5\lambda}{REN^{.081}} f_{so} & \varepsilon < \varepsilon_o \\ \\ \dfrac{2.5\lambda}{REN^{.081}} \left(\dfrac{(1-\varepsilon)}{(1-\varepsilon_o)} \dfrac{\varepsilon_o}{\varepsilon} \right)^{0.45} f_{so}, & \varepsilon \geq \varepsilon_o. \end{cases} \tag{L.10}$$

Defining $\phi_* = \phi - \rho A_B L / C,$ (L.11)

and

$$\frac{d^2 \ln \rho}{dt^2} = c_1(t) - \frac{g_o A_B^2 P_B}{V_F M_p} + \frac{g_o A_B^2 P_{res}}{V_F M_p},$$

where

$$c_1(t) = \ddot{m} \left(\frac{1}{m} - \frac{1}{\rho_p V_F} \right) + \frac{\dot{V}_F^2}{V_F^2} - \frac{\dot{m}^2}{m^2},$$

and

$$V_F = V_o + A_B(z_p - z_{po}) - \frac{c}{\rho_p} + \frac{m}{\rho_p},$$

$$\dot{m} = \rho_p S \frac{dx}{dt}, \qquad (L.12)$$

$$\ddot{m} = \rho_p \dot{S} \frac{dx}{dt} + \rho_p S \frac{d^2x}{dt^2}$$

$$= \rho_p \frac{dS}{dx} \left(\frac{dx}{dt}\right)^2 + \rho_p S \frac{d^2x}{dt^2}, \qquad (L.13)$$

with

$$\frac{dx}{dt} = aP_m^n \qquad (L.14)$$

and

$$\frac{d^2x}{dt^2} = anP_m^{n-1} \frac{dP_m}{dt}, \qquad (L.15)$$

where dP_m/dt is determined numerically, or from

$$\frac{dP_m}{dt} = \frac{\frac{\dot{m}RT}{mw} - P_m\dot{V} + \frac{\dot{T}Rm}{mw}}{V}$$

and

$$\dot{V}_p = \frac{g_o A_B}{M_p} (P_B - P_{res}). \qquad (L.16)$$

Then

$$P(z) = P_{Br} - \frac{(k_{11}P_B + k_{12})}{2} z^2, \qquad (L.17)$$

where

$$k_{11} = \frac{C \frac{\phi_o}{\varepsilon} \left(1 - \frac{LA_B}{V_F}\right) \frac{A_B}{M_p} k_2}{z_b V(z_b)} \qquad (L.18)$$

and

$$k_2 = \frac{1}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \qquad (L.19)$$

and

$$k_{12} = \frac{\phi_1' C k_2}{g_o z_b V(z_b)} - k_{11} \quad P_{res}, \tag{L.20}$$

$$\phi_1' = \dot{\phi}_* U_g - \dot{\phi} U_p - \frac{\phi_* \dot{\varepsilon}}{\varepsilon^2} \left(V_p + L \frac{d\ln\rho}{dt} - U_p \right)$$

$$+ \frac{\phi_* \dot{L}}{\varepsilon} \frac{d\ln\rho}{dt} + \frac{2\phi_* U_g}{z_b} (U_g - U_p)$$

$$+ \phi_2 \frac{\rho}{D_p \rho_p} (U_g - U_p)^2 f_{sc} + \frac{L\phi_* C_1(t)}{\varepsilon}, \tag{L.21}$$

and

$$\phi_2 = 1 - \phi - \phi_* \frac{(1-\varepsilon)}{\varepsilon}. \tag{L.22}$$

For the ullage region, the momentum equation for the gas is

$$\frac{\partial P}{\partial z} = - \frac{\rho}{g_o} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right), \tag{L.23}$$

and, therefore, the pressure in the ullage region is

$$P(z) = P(z_b) - \frac{\rho}{g_o} \left[(\dot{V}_p + z_p \Delta)(z - z_b) - (z^2 - z_b^2) \frac{\Delta}{2} \right], \tag{L.24}$$

where

$$\Delta = \frac{d^2 \ln\rho}{dt^2} - \left(\frac{d\ln\rho}{dt} \right)^2.$$

Defining

$$P_m = \frac{\int_o^{z_p} P(z) A(z) dz}{\int_o^{z_p} A(z) dz}, \tag{L.25}$$

then

$$P_B = \frac{P_m - D}{G} \tag{L.26}$$

and

$$P_{Br} = AP_B + B, \tag{L.27}$$

where

$$D = \frac{k_{12}z_b^2}{2} - \frac{\rho L A_B P_{res}}{M_p} - \frac{k_{12}z_b^2}{2z_p}\left(\frac{z_b}{3}+L\right) + \frac{\rho L^2 A_B P_{res}}{2z_p M_p}$$
$$+ \frac{\rho L^2}{2g_o}\left(1-\frac{2L}{3z_p}\right)\left(c_1(t)-\left(\frac{dln\rho}{dt}\right)^2\right)$$
$$+ \frac{A_B^2 \rho L^2}{2M_p V_F}\left(1-\frac{2L}{3z_p}\right) P_{re}, \tag{L.28}$$

$$G = 1 + \frac{k_{11}z_b^2}{2} + \frac{\rho L A_B}{M_p} - \frac{A_B^2 \rho L^2}{2V_F M_p}\left(1-\frac{2L}{3z_p}\right) - \frac{k_{11}z_b^3}{6z_p}$$
$$- \frac{k_{11}Lz_b^2}{2z_p} - \frac{\rho L^2 A_B}{2z_p M_p}, \tag{L.29}$$

$$A = 1 + \frac{k_{11}z_b^2}{2} + \frac{\rho L A_B}{M_p} - \frac{A_B^2 \rho L^2}{2V_F M_p}, \tag{L.30}$$

and

$$B = \frac{k_{12}z_b^2}{2} - \frac{\rho L A_B}{M_p} P_{res}$$
$$+ \frac{\rho L^2}{2g_o}\left(c_1(t) - \left(\frac{dln\rho}{dt}\right)^2\right) + \frac{A_B^2 \rho L^2}{2V_F M_p} P_{res}. \tag{L.31}$$

The kinetic energy of the propellant and gases is

$$KE = \frac{A_B z_b}{6g_o}\left[\varepsilon\rho U_g^2 + (1-\varepsilon)\rho_p U_p^2\right]$$
$$+ \frac{\rho A_B L}{6g_o}\left(3V_p^2 + 3V_p L \frac{dln\rho}{dt} + L^2\left(\frac{dln\rho}{dt}\right)^2\right).$$

The mean pressure over the mixture region is

$$P_{mx} = P_{Br} - \left(\frac{k_{11}P_B + k_{12}}{6}\right) z_b^2.$$

1.4 Combined Influence of Chambrage and Propellant Velocity Lag. The Robbins-Gough-Anderson (RGA) gradient equation, which combines the influences of chambrage and multiphase flow, assumes that the variation in area is confined to the propellant chamber and that the propellant initially fills the chamber. The cross-sectional area within the ullage is accordingly uniform and equal to A_B at all points. Nomenclature is as in the previous gradient equations.

The continuity equations for the mixture region are

$$\frac{\partial \varepsilon \rho}{\partial t} + \frac{1}{A}\frac{\partial \varepsilon \rho u A}{\partial z} = \dot{m}(t) \tag{R.1}$$

$$\rho_p \frac{\partial \varepsilon}{\partial t} - \frac{1}{A}\frac{\partial (1-\varepsilon)\rho_p u_p A}{\partial z} = \dot{m}(t). \tag{R.2}$$

Making assumptions analogous to the Lagrange assumption, namely

$$\frac{\partial \rho}{\partial z} = \frac{\partial \varepsilon}{\partial z} = 0 \quad \textit{in the mixture region} \tag{R.3}$$

and

$$\frac{\partial \rho}{\partial z} = 0 \quad \textit{in the ullage region} \tag{R.4}$$

and noting the assumption $\dot{m}(t)$ is only a function of time, then

$$u_p = \frac{U_p A_B V(z)}{V(z_b) A(z)} \tag{R.5}$$

and

$$u = \frac{U_g A_B V(z)}{V(z_b) A(z)}. \tag{R.6}$$

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form

$$\varepsilon \ (U_g - U_p) = U_{g+} - U_p$$

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption R.3.

Consider the momentum equations for the mixture region,

$$\frac{1}{A}\left[\frac{\partial A\varepsilon\rho u}{\partial t} + \frac{\partial A\varepsilon\rho u^2}{\partial z}\right] + \varepsilon g_o \frac{\partial P}{\partial z} = -f_s + \dot{m}u_p \qquad (R.7)$$

and

$$\frac{1}{A}\left[\frac{\partial A(1-\varepsilon)\rho_p u_p}{\partial t} + \frac{\partial A(1-\varepsilon)\rho_p u_p^2}{\partial z}\right] + (1-\varepsilon)g_o \frac{\partial P}{\partial z} = f_s - \dot{m}u_p, \qquad (R.8)$$

where

$$f_s = \frac{(1-\varepsilon)}{D_p}\ \rho\ (u-u_p)^2\ f_{sc} \qquad (R.9)$$

and for

$$REN = \frac{\rho D_p[U_g - U_p]}{\mu}$$

and for

$$\lambda = \left[\frac{0.5 + \frac{GL}{GD}}{\left(1.5\ \frac{GL}{GD}\right)^{2/3}}\right]^{2.17}$$

and with ε_o the initial porosity

$$\varepsilon_o = 1 - \frac{C}{\rho_p V(z_{p_o})}.$$

Then

$$f_{sc} = \begin{cases} \dfrac{2.5\lambda}{REN^{.081}}\ f_{so} & \varepsilon < \varepsilon_o \\ \\ \dfrac{2.5\lambda}{REN^{.081}}\left(\dfrac{(1-\varepsilon)}{(1-\varepsilon_o)}\ \dfrac{\varepsilon_o}{\varepsilon}\right)^{0.45} f_{so}, & \varepsilon \geq \varepsilon_o. \end{cases}$$

Defining

$$\phi_* = \phi - \rho A_B L/C \qquad (R.10)$$

and

$$\frac{d^2 \ln\rho}{dt^2} = C_1(t) - \frac{g_o A_B^2 P_B}{V_F M_p} + \frac{g_o A_B^2 P_{res}}{M_p V_F}, \qquad (R.11)$$

where

$$C_1(t) = \ddot{m}\left(\frac{1}{m} - \frac{1}{\rho_p V_F}\right) + \frac{\dot{V}_F^2}{V_F^2} - \frac{\dot{m}^2}{m^2} \tag{R.12}$$

and

$$V_F = V_o + A_B\ (z_p - z_{po}) - \frac{C}{\rho_p} + \frac{m}{\rho_p},$$

$$\dot{m} = \rho_p\ S\ \frac{dx}{dt},$$

$$\ddot{m} = \rho_p\ \dot{S}\ \frac{dx}{dt} + \rho_p\ S\ \frac{d^2x}{dt^2}, \tag{R.13}$$

or

$$\ddot{m} = \rho_p\ \frac{dS}{dx}\left(\frac{dx}{dt}\right)^2 + \rho_p\ S\ \frac{d^2x}{dt^2}$$

with

$$\frac{dx}{dt} = aP_m^n \tag{R.14}$$

and

$$\frac{d^2x}{dt^2} = anP_m^{n-1}\ \frac{dP_m}{dt} \tag{R.15}$$

where dP_m/dt is determined numerically, or from

$$\frac{dP_m}{dt} = \frac{\frac{\dot{m}RT}{mw} - P_m\dot{V} + \frac{mR\dot{T}}{mw}}{V}$$

and

$$\dot{V}_p = \frac{g_o A_B}{M_p}\ (P_B - P_{res}). \tag{R.16}$$

Then

$$P(z) = P_{Br} + (a_1(t) + a_2(t)P_B)\ J_1(z) + b(t)J_2(z) \tag{R.17}$$

in the mixture region, where

$$a_1(t) = \frac{CA_B}{g_o V^2(z_b)}\left(\frac{\phi_3 A}{V(z_b)} - \frac{\phi'_1 + D - \frac{E\phi_4 g_o A_B}{\varepsilon M_p}\ P_{res}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}}\right) \tag{R.18}$$

and

$$a_2(t) = \frac{-\dfrac{CE\phi_* A_B^2 P_B}{V^2(z_b)\varepsilon M_p}}{1 - \dfrac{\phi_2 C}{\rho_p V(z_b)}} \qquad (R.19)$$

and

$$b(t) = -\frac{C\phi_3 A_B^2}{2g_o V^3(z_b)}, \qquad (R.20)$$

$$J_1(z) = \int_o^z \frac{V(z)}{A(z)}\, dz, \qquad (R.21)$$

$$J_2(z) = \frac{V^2(z)}{A^2(z)},$$

$$\phi'_1 = \dot{\phi}_* U_g - \dot{\phi} U_p - \frac{\phi_* \dot{\varepsilon}}{\varepsilon^2}\left[V_p + L\frac{d\ln\rho}{dt} - U_p\right] + \frac{L\phi_*}{\varepsilon}\frac{d\ln\rho}{dt}$$

$$+ \frac{2A_B\phi_* U_g}{V(z_b)}[U_g - U_p] + \phi_2 \frac{\rho}{\rho_p D_p}(U_g - U_p)^2 f_{sc}, \qquad (R.21a)$$

and

$$\phi_2 = 1 - \phi - \phi_* \frac{(1-\varepsilon)}{\varepsilon},$$

$$\phi_3 = \phi_* U_g^2 + (1-\phi) U_p^2,$$

$$D = \frac{L\phi_*}{\varepsilon} C_1(t),$$

$$E = 1 - \frac{LA_B}{V_F}. \qquad (R.22)$$

Also in the ullage region

$$\frac{\partial P}{\partial z} = -\frac{\rho}{g_o}\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z}\right] \qquad (R.23)$$

and

$$P(z) = P(z_b) - \frac{\rho}{g_o}\left[(\dot{V}_p + z_p\Delta)(z - z_b) - (z^2 - z_b^2)\frac{\Delta}{2}\right], \tag{R.24}$$

where

$$\Delta = \frac{d^2\ln\rho}{dt^2} - \left(\frac{d\ln\rho}{dt}\right)^2.$$

Defining the mean pressure as

$$P_m = \frac{\int_o^{z_p} P(z)A(z)dz}{\int_o^{z_p} A(z)dz} \tag{R.25}$$

and substituting for $\dot{V}_p$ and $d^2\ln\rho/dt^2$,

then

$$P_{Br} = P_B\left(1 - a_2(t)J_1(z_b) + \frac{\rho L A_B}{M_p} - \frac{\rho L^2 A_B^2}{2V_F M_p}\right)$$
$$+ \frac{\rho L^2}{2g_o}\left(C_1(t) - \left(\frac{d\ln\rho}{dt}\right)^2 + \frac{g_o A_B^2 P_{res}}{M_p V_F}\right)$$
$$- a_1(t)J_1(z_b) - b(t)J_2(z_b) - \frac{A_B\rho L P_{res}}{M_p} \tag{R.26}$$

and

$$P_B = \left[P_m - \frac{\rho L^2}{2g_o}\left(C_1(t) - \left(\frac{d\ln\rho}{dt}\right)^2 + \frac{g_o A_B^2 P_{res}}{M_p V_F}\right)\right.$$
$$- \frac{\rho A_B L^3}{3g_o V(z_p)}\left(C_1(t) + \frac{g_o A_B^2 P_{res}}{M_p V_F} - \left(\frac{d\ln\rho}{dt}\right)^2\right) + \frac{a_1(t)J_3(z_b)}{V(z_p)}$$
$$+ \frac{b(t)J_4(z_b)}{V(z_p)} + \frac{A_B L a_1(t)J_1(z_b)}{V(z_p)} + \frac{A_B L b(t)J_2(z_b)}{V(z_p)} + \frac{\rho A_B^2 L^2 P_{res}}{2V(z_p)M_p}$$
$$\left.- a_1(t)J_1(z_b) - b(t)J_2(z_b) - \frac{A_B\rho L P_{res}}{M_p}\right] /$$
$$\left(1 + \frac{A_B L a_2(t)J_1(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2}{2V(z_p)M_p} + \frac{a_2(t)J_3(z_b)}{V(z_p)} - a_2(t)J_1(z_b)\right.$$
$$\left.+ \frac{\rho L A_B}{M_p} - \frac{\rho L^2 A_B^2}{2V_F M_p} + \frac{\rho A_B L^3 A_B^2}{3V(z_p)V_F M_p}\right), \tag{R.27}$$

where

$$J_3(z_b) \;=\; \int_o^{z_b} A(z)J_1(z)dz \qquad (R.28)$$

and

$$J_4(z_b) \;=\; \int_o^{z_b} \frac{V^2(z)}{A^2(z)}\, dz. \qquad (R.29)$$

The kinetic energy is

$$KE \;=\; \left(\frac{1-\varepsilon}{2g_o}\right) \frac{U_p^2 A_B^2 \rho_p J_4(z_b)}{V(z_b)^2} + \frac{\varepsilon}{2g_o} \frac{U_g^2 A_B^2 \rho J_4(z_b)}{V(z_b)^2}$$

$$+ \frac{\rho A_B L}{6g_o}\left(3V_p^2 + 3V_p \rho L \frac{dln\rho}{dt} + L^2\left(\frac{dln\rho}{dt}\right)^2\right). \qquad (R.30)$$

The mean pressure over the mixture region is

$$P_{mix} \;=\; P_{Br} + \frac{(a_1(t) + a_2(t)P_B)}{V(z_b)} J_3(z_b) + \frac{b(t)J_4(z_b)}{V(z_b)}. \qquad (R.31)$$

2. CALCULATIONS

A number of propellant charges were simulated with XKTC to probe the influence of chambrage and velocity lag. The same charges were simulated with IBRGA (input description and listing given in Appendices 4 and 5) using data bases as consistent with those of XKTC as the physical scope of the lumped-parameter model would permit, and using each of the various pressure gradient models. The resultant maximum breech pressures, velocities, and histories of pressure-time and mean-pressure-to-base-pressure curves are compared.

The calculations performed with XKTC involved data bases with evenly distributed seven-perforated propellant having an initial porosity of 0.4, zero barrel resistance, and with all the propellant ignited at the initial instant. All calculations were performed for a flat-based projectile and nominal heat loss.

The parameters used in the computer codes were:

Large chamber simulations:

Bore diameter	127 mm
Volume	9,832.2 cm^3
Travel	4.572 m
Propellant mass	9.8 kg
Projectile mass	2.45, 9.8, 39.2 kg

Small chamber simulations:

Bore diameter	28.65 mm
Volume	98.322 cm^3
Travel	1.880 m
Propellant mass	.098 kg
Projectile mass	.0245, .098, .392 kg

Propellant characteristics:

Impetus	1,136 J/g
Covolume	.976 cm^3/g
Gamma	1.23
Flame temperature	3,141 K
Molecular weight	23.0 g/gmole
Density	1.66 g/cm^3
Burning rate	$1.10519p^{1.0}$ mm/s (p is in MPa)

Three maximum breech pressures were studied, nominally 517 MPa, 345 MPa, and 172 MPa. The maximum pressures were achieved by allowing the web to vary, with the grain length kept between two to three times the outer diameter. Calculations were performed for both bore diameter chambers and right circular cylindrical chambers with a larger diameter than the bore. Chambrage (defined as the distance over which the chamber tapers down to the bore area) was 76.2 mm for the large volume chamber, with the chamber length being changed from 776.22 mm (straight chamber) to 541.02 mm. For the smaller volume chamber, we had 25.4 mm of chambrage with the chamber length being changed from 152.4 mm (straight chamber) to 101.6 mm.

3. RESULTS

3.1 Chambrage. The effect of chambrage was assessed at a nominal pressure of 345 MPa using XKTC. Calculations were performed for a straight chamber to determine grain dimensions corresponding to a maximum breech pressure of 345 MPa. These grain dimensions were then used with a chamber with the same volume but having chambrage. The presence of chambrage was found to reduce the maximum breech pressure substantially. Finally, XKTC was run with chambrage and the dimensions of the grains changed to restore the maximum breech pressure to 345 MPa. These sets of grain dimensions were then used with a version of IBRGA into which the chambrage gradient equation had been encoded. Equivalent computer runs on IBRGA with the Lagrange gradient are also given in Table 1.

Adding chambrage to the gradient equation in the lumped-parameter code captures the large drop in maximum breech pressure and the concomitant drop in muzzle velocity seen when chambrage is added to XKTC. The gradient equation with chambrage also gives the proper amount of increase in maximum breech pressure and muzzle velocity, as seen in XKTC calculations, when the grain dimensions are changed to

TABLE 1. Chambrage Calculations.

			XKTC		Chambrage gradient		Lagrange gradient	
Chamber volume, cm^3	c/m	Chambrage, mm	Maximum breech pressure, MPa	Muzzle velocity, m/s	Maximum breech pressure, MPa	Muzzle velocity, m/s	Maximum breech pressure, MPa	Muzzle velocity, m/s
9,832.2	0.25	none	345	769	349	775	350	775
9,832.2	0.25	76.2	336	764	337	766		
9,832.2	0.25	76.2	345	774	346	776	358	784
9,832.2	1.0	none	345	1,352	347	1,363	347	1,363
9,832.2	1.0	76.2	310	1,303	307	1,308		
9,832.2	1.0	76.2	345	1,376	339	1,375	384	1,429
9,832.2	4.0	none	345	1,964	360	1,962	360	1,962
9,832.2	4.0	76.2	268	1,764	273	1,771		
9,832.2	4.0	76.2	345	1,964	339	1,990	452	2,184
98.322	0.25	none	345	895	348	880	348	880
98.322	0.25	25.4	337	895	334	872		
98.322	0.25	25.4	345	901	341	879	355	886
98.322	1.0	none	345	1,560	347	1,517	347	1,517
98.322	1.0	25.4	308	1,526	305	1,469		
98.322	1.0	25.4	345	1,595	340	1,536	387	1,582
98.322	4.0	none	345	2,298	371	2,198	371	2,198
98.322	4.0	25.4	275	2,149	280	2,028		
98.322	4.0	25.4	345	2,354	342	2,230	457	2,389

restore the maximum breech pressure to 345 MPa. If the Lagrange gradient equation were to be used, a difference from XKTC of approximately 10% at a charge mass to projectile mass ratio (c/m) of one and approximately 30% at a c/m of four would occur in the predicted values of maximum breech pressure.

The chambrage gradient equation gives calculated maximum breech pressures, which are as much as 7% high and muzzle velocities low by 3 to 4% when compared to XKTC calculations. The resultant gradient structure is illustrated, in Figures 1-6, with representative plots of the ratio of the mean pressure to the base pressure for both XKTC and the analytic chambrage gradient equation. The figures also include histories of breech and base pressure.

3.2 Multiphase Flow. The effects of multiphase flow were examined using XKTC. Calculations were based on a straight chamber; nominal maximum breech pressures of 172 MPa, 345 MPa, and 517 MPa; large and small chamber volumes; and values of c/m equal to .25, 1, and 4. The grain dimensions from the XKTC calculations were used to define equivalent data bases for IBRGA into which the two-phase gradient equation had been encoded.

The pressure at which the propellant burns is taken as the mean pressure over the mixture region instead of the mean pressure over the entire volume behind the projectile, as is conventionally assumed. Comparable Lagrange gradient calculations are also supplied in Table 2.

The calculated maximum breech pressures and velocities for both the two-phase and Lagrange gradient for c/m of .25 are in very close agreement with XKTC at all pressures. For calculations at c/m's of 1 and 4, the Lagrange and two phase gradients tend to give close maximum breech pressures, at worst about 6% different from XKTC.

The history of the ratio of space mean to base pressures for representative two-phase gradient calculations and for corresponding XKTC calculations is plotted in Figures 7-12, together with associated breech-pressure and base-pressure histories. The Lagrange gradient would be a straight line with a ratio value of 1.083 for a c/m of 0.25; 1.333 for a c/m of 1; and 2.333 for a c/m of 4. The two-phase gradient equation has the same shape and magnitude of the first peak in the mean pressure to base pressure ratio as XKTC, with the maximum occurring earlier in the analytic two-phase gradient calculations and the recovery to the Lagrange ratio value not being as fast as XKTC. The comparison of shape and magnitude of the pressure ratio histories strongly suggests that the dominant physical processes are captured by the two-phase gradient equation. In particular, the undulatory character of the curve is seen to be attributable, at least in part, to the two-phase aspects of the flow, rather than being due to transient or wave propagation phenomena as might at first be thought.

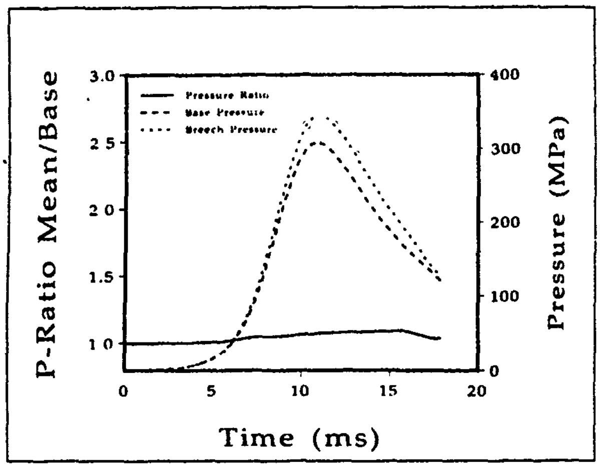

Figure 1. XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and 76.2 mm of Chambrage.

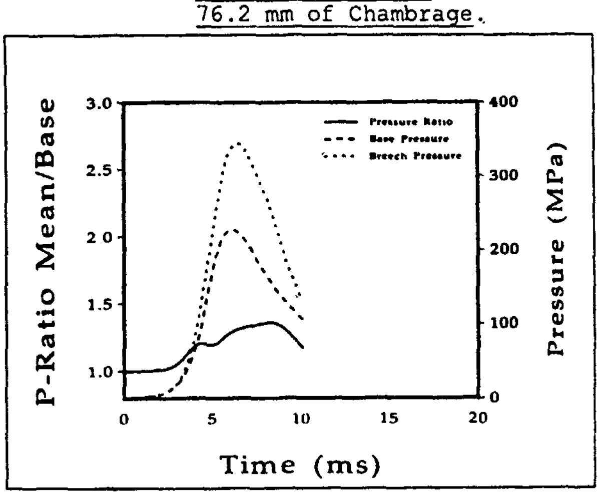

Figure 2. XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and 76.2 mm of Chambrage.

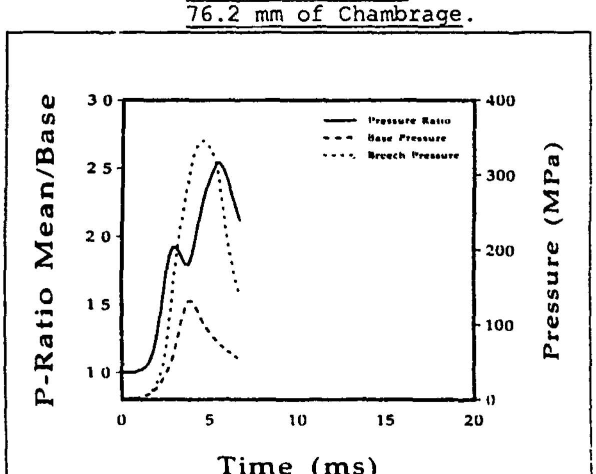

Figure 3. XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and 76.2 mm of Chambrage.

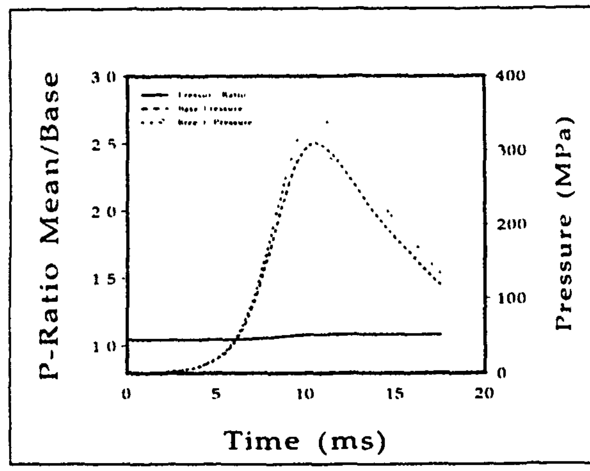

Figure 4. IBRGA Calculation With Chambrage Gradient With Figure 1 Values.

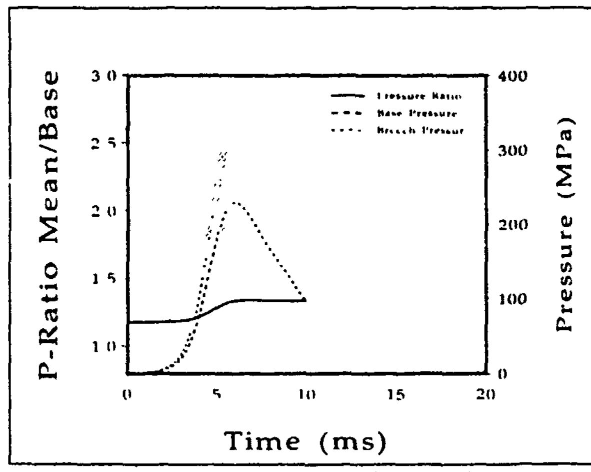

Figure 5. IBRGA Calculation With Chambrage Gradient With Figure 2 Values.

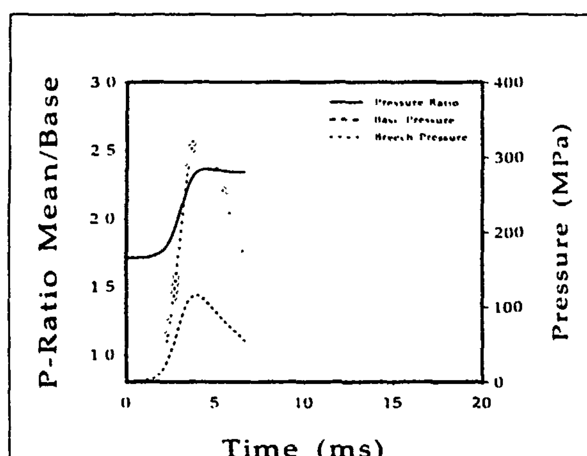

Figure 6. IBRGA Calculation With Chambrage Gradient With Figure 3 Values.

TABLE 2. Two-Phase Calculations.

Chamber volume, cm^3	c/m	XKTC Maximum breech pressure, MPa	XKTC Muzzle velocity, m/s	Two-Phase gradient Maximum breech pressure, MPa	Two-Phase gradient Muzzle velocity, m/s	Lagrange gradient Maximum breech pressure, MPa	Lagrange gradient Muzzle velocity, m/s
9,832.2	0.25	174	549	177	557	179	557
9,832.2	0.25	345	769	348	773	350	775
9,832.2	0.25	518	887	521	891	519	891
9,832.2	1.0	173	957	177	970	184	966
9,832.2	1.0	345	1,352	343	1,355	347	1,363
9,832.2	1.0	516	1,593	498	1,556	497	1,560
9,832.2	4.0	172	1,389	172	1,380	184	1,354
9,832.2	4.0	345	1,964	351	1,930	360	1,962
9,823.2	4.0	518	2,391	524	2,290	528	2,306
98.322	0.25	172	641	177	632	179	630
98.322	0.25	345	895	346	880	348	880
98.322	0.25	518	987	514	974	513	974
98.322	1.0	172	1,130	180	1,107	186	1,094
98.322	1.0	345	1,560	343	1,516	347	1,517
98.322	1.0	516	1,756	510	1,703	512	1,705
98.322	4.0	172	1,682	174	1,581	185	1,524
98.322	4.0	345	2,298	360	2,188	371	2,198
98.322	4.0	518	2,685	545	2,516	552	2,527

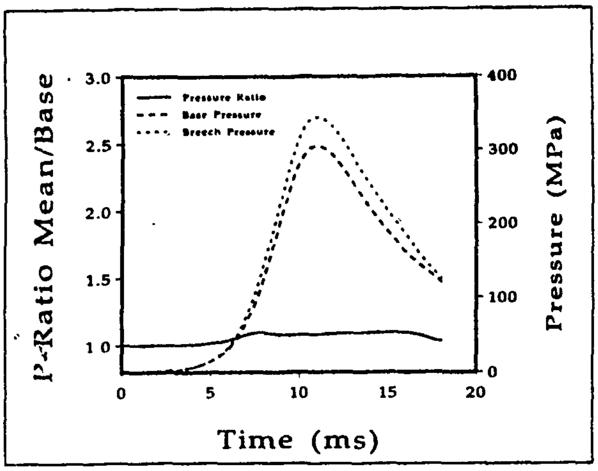

Figure 7. XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and No Chambrage.

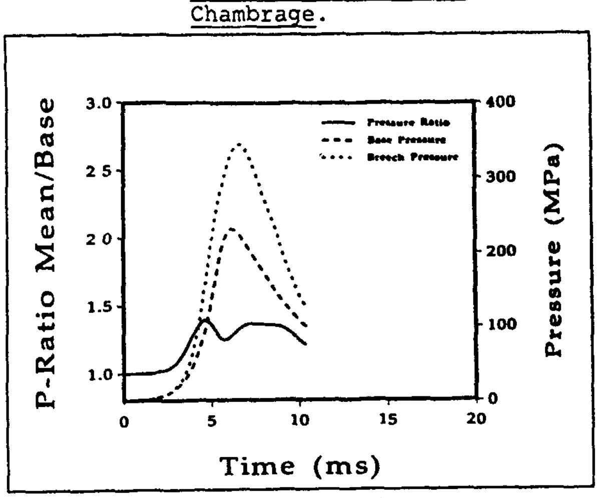

Figure 8. XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and No Chambrage.

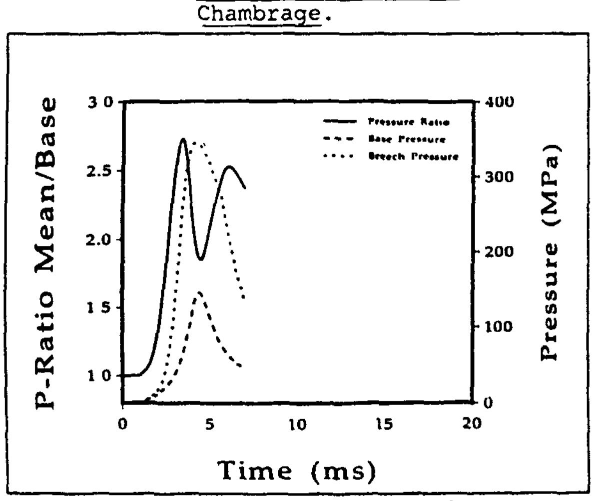

Figure 9. XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and No Chambrage.

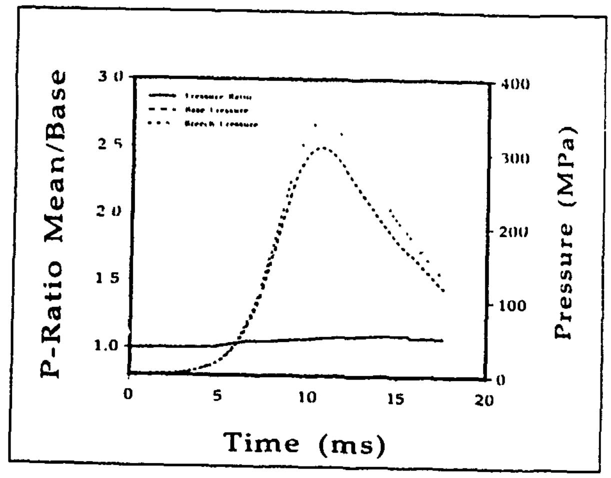

Figure 10. IBRGA Calculation With Two-Phase Gradient With Figure 7 Values.

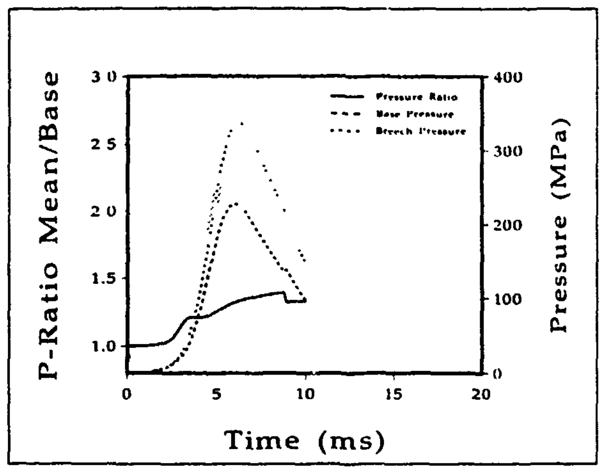

Figure 11. IBRGA Calculation With Two-Phase Gradient With Figure 8 Values.

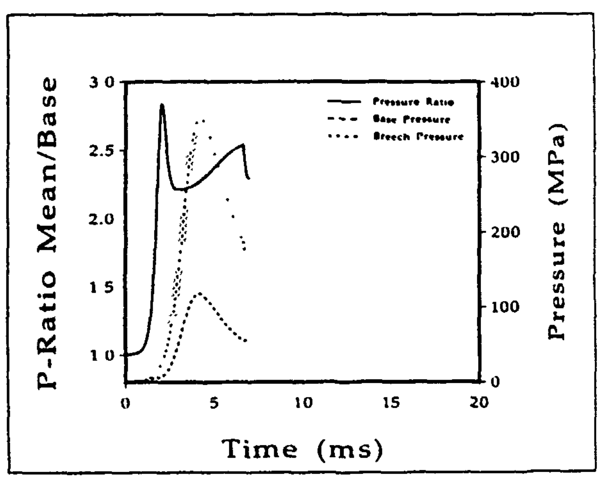

Figure 12. IBRGA Calculation With Two-Phase Gradient With Figure 9 Values.

The dropoff of the pressure ratio at the slivering event seen in IBRGA calculations may cause numerical problems. (This event is smoothed over in XKTC since all the grains do not sliver at the same time because of the different pressure fields seen by the grains, as well as the finite time required for the transfer of information.) This problem is not felt to affect the calculated velocity significantly, but a smoothing routine has been incorporated into IBRGA to circumvent any difficulty.

The two-phase gradient equation also has, as input, a multiplier to the friction factor. By making this value small, an approximation to stick propellant behavior can be captured.

3.3 Combined. Table 1 is reproduced in Table 3 with the chambrage gradient equation replaced by the RGA gradient equation with the appropriate kinetic energy term. As with the two-phase gradient equation, the burning rate is calculated from mean pressure over the mixture region.

The shapes of the ratio and pressure-time plots, Figures 13-18, are qualitatively similar to those produced by XKTC. This reinforces the idea that both two phase and chambrage effects are important factors in respect to the time-dependent behavior of the pressure gradient. The result of the RGA gradient equation calculations are in general slightly better, when compared to XKTC, than the corresponding calculations with the chambrage gradient equation.

One of the RGA gradient equation inputs is a multiplier to the friction factor. By making this value small, an approximation to stick propellant behavior can be captured.

4. CONCLUSIONS

It is concluded that incorporating chambrage into a gradient equation is important and can make as much as a 20 to 30% difference in predicted maximum breech pressure at c/m of 4. The inclusion of two-phase effects into the gradient equation seems to be well motivated in that a large portion of the structure of the gradient history is captured.

At low c/m's (0.25), any of the gradient equations will give good results; but as the c/m gets larger (>1), the correction for chambrage is required.

The use of the RGA gradient equation offers a natural way to account for stick propellant phenomena having to do with stick propellant motion and the pressure history in which it burns.

The use of the proper kinetic energy term for the gradient equation, as well as the calculation of the mean pressure over the mixture region, should be included in any gradient model when applicable.

TABLE 3. RGA Calculations.

			XKTC		RGA gradient		Lagrange gradient	
Chamber volume, cm^3	c/m	Chambrage, mm	Maximum breech pressure, MPa	Muzzle velocity, m/s	Maximum breech pressure, MPa	Muzzle velocity, m/s	Maximum breech pressure, MPa	Muzzle velocity, m/s
9,832.2	0.25	none	345	769	348	773	350	775
9,832.2	0.25	76.2	336	764	336	766		
9,832.2	0.25	76.2	345	774	345	775	358	784
9,832.2	1.0	none	345	1,352	343	1,355	347	1,363
9,832.2	1.0	76.2	310	1,303	304	1,300		
9,832.2	1.0	76.2	345	1,376	337	1,369	384	1,429
9,832.2	4.0	none	345	1,964	351	1,930	360	1,961
9,832.2	4.0	76.2	268	1,764	266	1,747		
9,832.2	4.0	76.2	345	1,964	334	1,964	452	2,183
98.322	0.25	none	345	895	346	879	348	880
98.322	0.25	25.4	337	895	332	872		
98.322	0.25	25.4	345	901	340	878	355	886
98.322	1.0	none	345	1,560	342	1,515	347	1,517
98.322	1.0	25.4	308	1,526	302	1,468		
98.322	1.0	25.4	345	1,595	337	1,535	387	1,582
98.322	4.0	none	345	2,298	361	2,190	371	2,198
98.322	4.0	25.4	275	2,149	272	2,023		
98.322	4.0	25.4	345	2,354	336	2,222	457	2,389

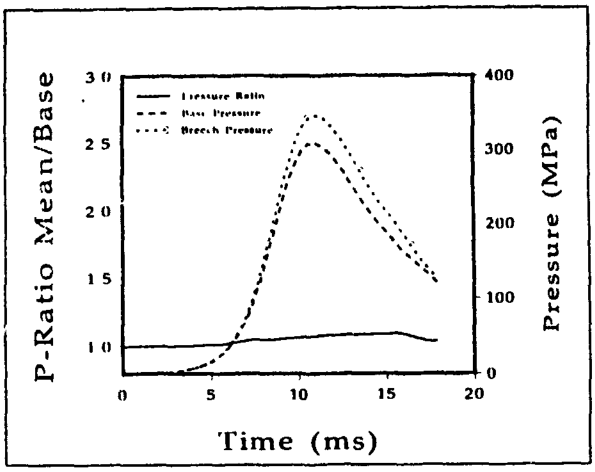

Figure 13. XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and 76.2 mm of Chambrage.

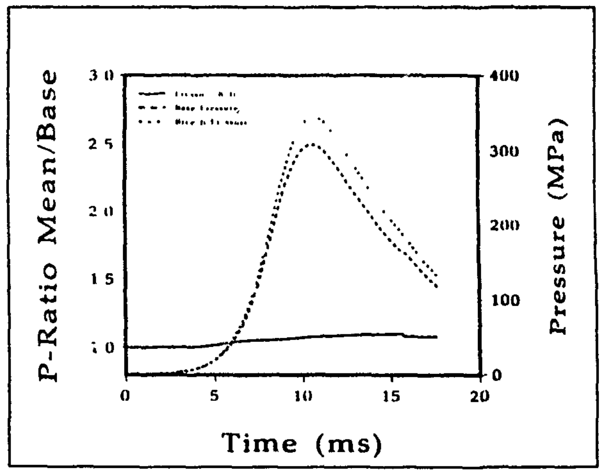

Figure 16. IBRGA Calculation With RGA Gradient With Figure 13 Values.

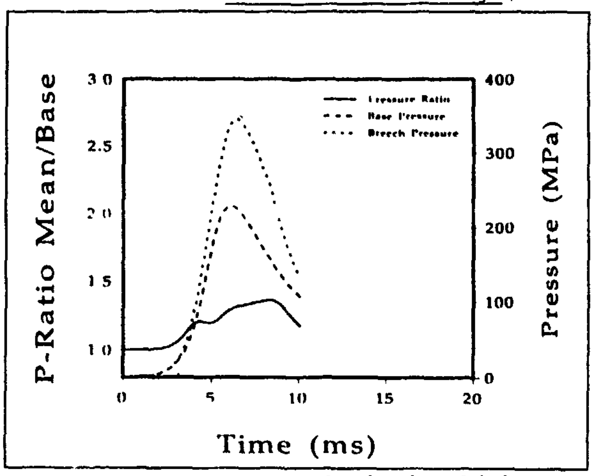

Figure 14. XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and 76.2 mm of Chambrage.

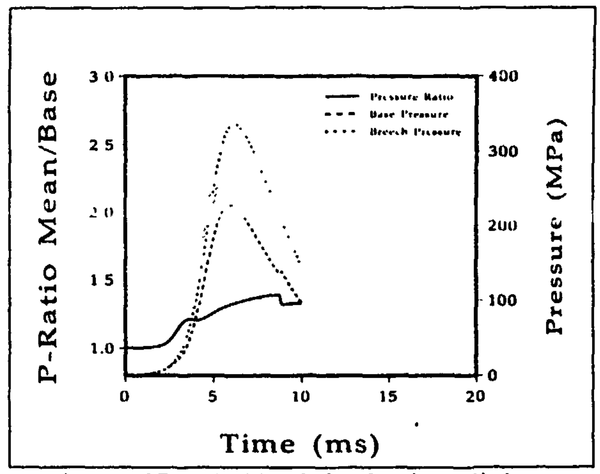

Figure 17. IBRGA Calculation With RGA Gradient With Figure 14 Values.

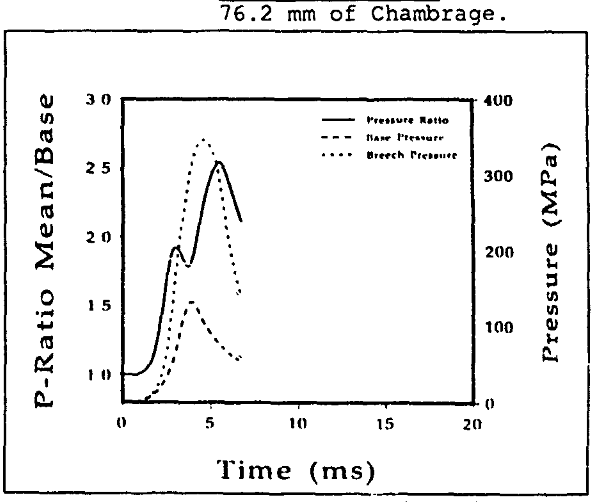

Figure 15. XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and 76.2 mm of Chambrage.

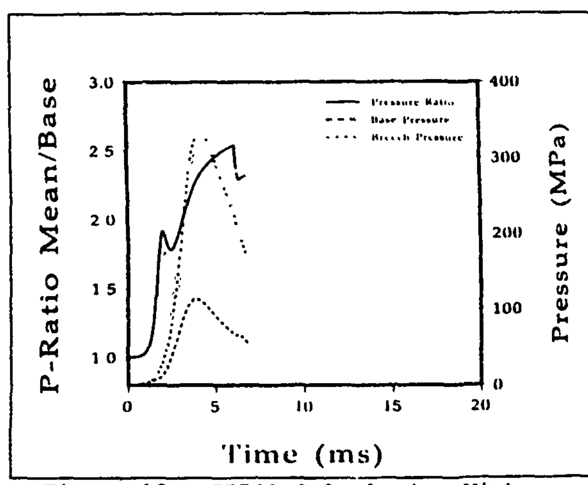

Figure 18. IBRGA Calculation With RGA Gradient With Figure 15 Values.

INTENTIONALLY LEFT BLANK.

5. REFERENCES

Anderson, R.D., and K.D. Fickie, "IBHVG2 -- A User's Guide," BRL-TR-2829, USA ARRADCOM, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, July 1987.

Gough, P.S. Contractor Report, DAAK11-85-D-0002, in preparation.

Gough, P.S. "The NOVA Code: A User's Manual." Indian Head Contract Report IHCR80-8, Naval Ordnance Station, Indian Head, MD, 1980.

Morrison, W.F., and G.P. Wren. "A Lumped Parameter Description of Liquid Injection in a Regenerative Liquid Propellant Gun." Proceedings of the 23rd JANNAF Combustion Meeting, CPIA Publication 457, Vol. 2, p. 464-489, October 1988.

Robbins, F.W. "Comparison of TDNOVA results with an Analytic Solution." ARBRL-MR-03299, USA ARRADCOM, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, July 1983.

Robbins, F.W. "Studies Supporting Development of a Modified Gradient Equation for Lumped Parameter Interior Ballistic Codes." Proceedings of the 12th Meeting of The Technical Cooperation Program, Vol. 5, October 1986.

Robbins, F.W., and A.W. Horst. "Detailed Characterization of the Interior Ballistics of Slotted Stick Propellant." BRL-TR-2591, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, September 1984.

Robbins, F.W., A.A. Koszoru, and T.C. Minor. "A Theoretical and Experimental Interior Ballistic Characterization of Combustible Cases." Proceedings of the 9th International Symposium on Ballistics, Part 1, p. 21-28, April 1986.

Vinti, J.P. "The Equations of Interior Ballistics." BRL Report 307, Ballistic Research Laboratories, Aberdeen Proving Ground, MD, October 1942.

INTENTIONALLY LEFT BLANK.

6. LIST OF SYMBOLS

ε, porosity defined as the volume of gas/total volume

λ, particle shape factor for gas flow resistance

μ, viscosity of gas

ρ, density

ρ_p, propellant density

ϕ, mass fraction burned

$A(z)$, area at position z

A_B, tube area

C, total charge mass

D_p, effective diameter of the propellant grain, i.e., 6* volume of propellant grain/surface area of a propellant grain

f_s, interphase drag

f_{so}, baseline friction factor

f_{sc}, dimensionless friction factor, a function of Reynolds number and particle shape

GD, grain diameter

GL, grain length

g_0, a constant to reconcile dimensions

L, the distance between the leading edge of the propellant bed and the base of the projectile, i.e., $L = (z_p - z_b)$

$\dot{m}(t)$, rate of combustion of propellant

m, mass of propellant burned

M_p, mass of projectile

m_w, molecular weight of propellant gas

P, pressure

P_B, projectile base pressure

P_{Br}, breech pressure

P_m, mean pressure

P_{res}, resistive pressure

R, universal gas constant

REN, particle reynolds number

S, surface area of the propellant

T, gas temperature

t, time

u, velocity

U_g, gas velocity on the mixture side at the leading edge of the propellant bed, i.e., $u(z_b)$

U_{g+}, gas velocity on the ullage side of the leading edge of the propellant bed

u_p, propellant velocity

U_p, velocity of the leading edge of the propellant bed, i.e., $u_p(z_b)$

$V(z)$, volume between the breech face and position z

V_F, free volume for gases to expand into

V_o, initial empty volume of chamber

V_p, velocity of the projectile

x, distance burned into the propellant

z, axial position measured from the breech face

z_b, position of the leading edge of the propellant bed

z_p, position of the base of the projectile

z_{po}, initial position of the base of the projectile

$\dot{}$, derivative with respect to time

$\ddot{}$, second derivative with respect to time

APPENDIX 1:
INFLUENCE OF CHAMBRAGE

INTENTIONALLY LEFT BLANK.

The following, is work by Gough (Gough, in preparation) with modifications and elaborations by Robbins.

Influence of Chambrage. For the effects of chambrage on the form of the gradient equation, the propellant is assumed to be uniformly distributed between the breech and the base of the projectile, and all variations in tube area arc confined to the chamber.

The continuity and momentum cquations for unsteady flow of a homogeneous inviscid substance through a tube with variable area are

$$\frac{\partial \rho}{\partial t} + \frac{1}{A}\left(\frac{\partial \rho A u}{\partial z}\right) = 0 \tag{C.1}$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + g_o \frac{\partial P}{\partial z} = 0. \tag{C.2}$$

Making the Lagrange assumption

$$\frac{\partial \rho}{\partial z} = 0, \tag{C.3}$$

then C.1 becomes

$$\frac{\partial \rho}{\partial t} = -\frac{\rho}{A}\left(\frac{\partial A u}{\partial z}\right). \tag{C.4}$$

The density may be written as

$$\rho = \frac{C}{V(z_p)}. \tag{C.5}$$

Differentiating (C.5), we get

$$\frac{d\rho}{dt} = -\left(\frac{C}{V^2(z_p)}\right)\left(\frac{dV(z_p)}{dt}\right). \tag{C.6}$$

But,

$$V(z_p) = V(z_{po}) + (z_p - z_{po})\ A_B\ , \tag{C.7}$$

and

$$\frac{dV(z_p)}{dt} = A_B \dot{z}_p = A_B V_p\ , \tag{C.8}$$

therefore,

$$\frac{d\rho}{dt} = -\frac{CA_BV_p}{V^2(z_p)}. \qquad (C.9)$$

But,

$$\rho = \frac{C}{V(z_p)},$$

therefore,

$$\frac{d\rho}{dt} = -\frac{A_B\rho V_p}{V(z_p)}. \qquad (C.10)$$

Substituting (C.10) into (C.4), we get

$$-\frac{A_B\rho V_p}{V(z_p)} = -\frac{\rho}{A}\left(\frac{\partial Au}{\partial z}\right) \qquad (C.11)$$

or

$$\frac{A_BAV_p}{V(z_p)} = \frac{\partial Au}{\partial z}. \qquad (C.12)$$

Integrating (C.12) with u(0) = 0 and noting that V_p and V (z_p) are functions of time only,

$$A(z)u(z) = Au = \frac{A_BV_p}{V(z_p)}\int_o^z A(z)dz = \frac{A_BV_pV(z)}{V(z_p)}. \qquad (C.13)$$

Therefore, the velocity u at a given time is given by:

$$u = \left(\frac{A_BV_p}{V(z_p)}\right)\left(\frac{V(z)}{A(z)}\right). \qquad (C.14)$$

Note that if A = A_B, then V(z) = A_Bz, then u = (z/z_p) V_p as for the Lagrange gradient. (C.15)

The pressure distribution follows from the momentum equation (C.2)

$$g_o\frac{\partial P}{\partial z} = -\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z}\right). \qquad (C.16)$$

Differentiating (C.14), we get $\partial u/\partial t$ and $\partial u/\partial z$ for substitution into (C.16).

To get $\partial u/\partial t$ we differentiate (C.14), noting that only V_p and $V(z_p)$ are functions of time and

$$\frac{\partial V(z_p)}{\partial t} = \frac{\partial (V(z_{po}) + (z_p - z_{po})\ A_B)}{\partial t} = A_B \dot{z}_p = A_B V_p, \tag{C.17}$$

then

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{A_B \dot{V}_p V(z)}{A(z)V(z_p)} - \left(\frac{A_B V_p V(z)}{A(z)V^2(z_p)}\right)\left(\frac{\partial V(z_p)}{\partial t}\right) \\ &= \frac{A_B \dot{V}_p V(z)}{A(z)V(z_p)} - \frac{A_B^2 V_p^2 V(z)}{A(z)V^2(z_p)}. \end{aligned} \tag{C.18}$$

To get $\partial u/\partial z$, we differentiate (C.14), noting that only V(z) and A(z) are functions of z and

$$\frac{\partial V(z)}{\partial z} = \frac{\partial \int_o^z A(z)dz}{\partial z} = A(z), \tag{C.19}$$

then

$$\begin{aligned} \frac{\partial u}{\partial z} &= \frac{A_B V_p A(z)}{V(z_p)A(z)} - \left(\frac{A_B V_p V(z)}{A^2(z)V(z_p)}\right)\left(\frac{dA}{dz}\right) \\ &= \frac{A_B V_p}{V(z_p)} - \left(\frac{A_B V_p V(z)}{A^2(z)V(z_p)}\right)\left(\frac{dA}{dz}\right). \end{aligned} \tag{C.20}$$

Substituting (C.5), (C.14), (C.18), and (C.20) into (C.16) yields

$$g_o\ \frac{\partial P}{\partial z} = -\ \frac{CA_B V(z)\dot{V}_p}{A(z)V^2(z_p)} + \left(\frac{CV^2(z)V_p^2 A_B^2}{A^3(z)V^3(z_p)}\right)\left(\frac{\partial A}{\partial z}\right). \tag{C.21}$$

Integrating (C.21), and noting $\dot{V}_p$ and $V(z_p)$ are functions only of time, then

$$\begin{aligned} g_o \int_o^z \partial P &= g_o(P(z) - P(0)) \\ &= -\ \frac{CA_B\dot{V}_p}{V^2(z_p)} \int_o^z \frac{V(z)dz}{A(z)} \\ &\quad + \frac{CV_p^2 A_B^2}{V^3(z_p)} \int_o^z \left(\frac{V^2(z)}{A^3(z)}\right)\left(\frac{\partial A}{\partial z}\right) dz. \end{aligned} \tag{C.22}$$

Noting

$$\int_o^z \left(\frac{V^2(z)}{A^3(z)}\right)\left(\frac{\partial A}{\partial z}\right) dz = \int_o^z \frac{V(z)}{A(z)} dz - \frac{V^2(z)}{2A^2(z)}$$

(from integration by parts), then (C.22), the pressure distribution behind the projectile is

$$g_o \ (P(z) - P(0)) = \left(\frac{CA_B^2V_p^2}{V^3(z_p)} - \frac{CA_B\dot{V}_p}{V^2(z_p)}\right) J_1(z)$$

$$- \frac{CV_p^2A_B^2}{2V^3(z_p)} J_2(z), \tag{C.23}$$

where

$$J_1(z) = \int_o^z \frac{V(z)}{A(z)} dz, \tag{C.24}$$

which can be evaluated algebraically or numerically if V(z) and A(z) are known, and

$$J_2(z) = \frac{V^2(z)}{A^2(z)}. \tag{C.25}$$

Rewriting (C.23) in the form

$$P(z) = P(0) + a(t)J_1(z) + b(t)J_2(z) \tag{C.26}$$

gives

$$a(t) = \frac{1}{g_o}\left(\frac{CA_B^2V_p^2}{V^3(z_p)} - \frac{CA_B\dot{V}_p}{V^2(z_p)}\right) \tag{C.27}$$

and

$$b(t) = -\frac{CV_p^2A_B^2}{2g_oV^3(z_p)}. \tag{C.28}$$

Defining the mean pressure in the chamber to be

$$P_m = \frac{\int_o^{z_p} P(z)A(z)dz}{\int_o^{z_p} A(z)dz}, \tag{C.29}$$

then using (C.26),

$$P_m = \frac{\int_o^{z_p} [P(0) + a(t)J_1(z) + b(t)J_2(z)] \; A(z)dz}{\int_o^{z_p} A(z)dz} \tag{C.30}$$

$$\begin{aligned} P_m &= \frac{P(0) \int_o^{z_p} A(z)dz}{V(z_p)} \\ &+ \frac{a(t) \int_c^{z_p} J_1(z)A(z)dz}{V(z_p)} \\ &+ \frac{b(t) \int_o^{z_p} J_2(z)A(z)dz}{V(z_p)} \end{aligned} \tag{C.31}$$

or

$$P_m = P(0) + \frac{a(t)}{V(z_p)} J_3(z_p) + \frac{b(t)}{V(z_p)} J_4(z_p) \tag{C.32}$$

where

$$\begin{aligned} J_3(z_p) &= \int_o^{z_p} A(z)J_1(z)dz \\ &= \int_o^{z_p} \left(\int_o^{z} \frac{V(x)}{A(x)} \, dx \right) A(z)dz \end{aligned} \tag{C.33}$$

and

$$\begin{aligned} J_4(z_p) &= \int_o^{z_p} A(z)J_2(z)dz \\ &= \int_o^{z_p} \frac{V^2(z)}{A(z)} \, dz, \end{aligned} \tag{C.34}$$

and where, if A(z) and V(z) are known, $J_3(z_p)$ and $J_4(z_p)$ may be evaluated algebraically or numerically. The acceleration of the projectile is given by

$$\dot{V}_p = \frac{g_o A_B}{M_p} [P_B - P_{res}]. \tag{C.35}$$

Substitution into (C.27) gives

$$a(t) = a_1(t) + a_2(t)P_B \tag{C.36}$$

where

$$a_1(t) = \frac{CA_B}{g_o V^2(z_p)} \left(\frac{A_B V_p^2}{V(z_p)} + \frac{g_o A_B P_{res}}{M_p} \right) \tag{C.37}$$

and

$$a_2(t) = -\frac{CA_B^2}{M_p V^2(z_p)} \quad . \tag{C.38}$$

Substituting (C.36) into (C.26) and (C.32) and noting $P_{Br} = P(0)$,

then

$$P_{Br} = [1 - a_2(t)J_1(z_p)]P_B - a_1(t)J_1(z_p) - b(t)J_2(z_p) \tag{C.39}$$

and

$$P_m = P_{Br} + \frac{[a_1(t) + a_2(t)P_B]}{V(z_p)} J_3(z_p) + \frac{b(t)}{V(z_p)} J_4(z_p). \tag{C.40}$$

Substituting (C.39) into (C.40) and rearranging terms,

$$\begin{aligned} P_m = & \left(1 - a_2(t)J_1(z_p) + \frac{a_2(t)J_3(z_p)}{V(z_p)}\right) P_B - a_1(t)J_1(z_p) \\ & - b(t)J_2(z_p) + \frac{a_1(t)J_3(z_p)}{V(z_p)} + \frac{b(t)J_4(z_p)}{V(z_p)}. \end{aligned} \tag{C.41}$$

Solving (C.41), we get P_B in terms of P_m

$$P_B = \frac{P_m + a_1(t)J_1(z_p) + b(t)J_2(z_p) - \frac{a_1(t)J_3(z_p)}{V(z_p)} - \frac{b(t)J_4(z_p)}{V(z_p)}}{1 - a_2(t)J_1(z_p) + \frac{a_2(t)J_3(z_p)}{V(z_p)}}. \tag{C.42}$$

Therefore, assuming the mean pressure, P_m, can be calculated from an equation of state, then (C.42) will give the projectile base pressure, P_B. The breech pressure, P_{Br}, is calculated from (C.39) knowing P_B. The pressure distribution is given by (C.26).

The evaluation of $J_1(z_p)$, $J_2(z_p)$, $J_3(z_p)$, and $J_4(z_p)$ can be simplified by noting that the variation in area is confined to the chamber and, therefore,

$$V(z) = V(z_{po}) + A_B(z - z_{po}), \quad for \quad z \geq z_{po}, \tag{C.43}$$

and

$$\begin{aligned} J_1(z_p) &= \int_o^{z_p} \frac{V(z)}{A(z)}\, dz = \int_o^{z_{po}} \frac{V(z)}{A(z)}\, dz + \int_{z_{po}}^{z_p} \frac{V(z)}{A(z)}\, dz \\ &= J_1(z_{po}) + \int_{z_{po}}^{z_p} \frac{V(z_{po}) + A_B(z - z_{po})}{A_B}\, dz \\ &= J_1(z_{po}) + \frac{1}{A_B}\left(V(z_{po})\ (z_p - z_{po}) + \frac{A_B(z_p - z_{po})^2}{2}\right) \end{aligned} \tag{C.44}$$

$$J_2(z_p) = \frac{V^2(z)}{A^2(z)}\Big|_{z_p} = \frac{[V(z_{po}) + A_B(z_p - z_{po})]^2}{A_B^2}, \tag{C.45}$$

$$\begin{aligned} J_3(z_p) &= \int_o^{z_p} \left(\int_o^{z} \frac{V(x)}{A(x)}\, dx\right) A(z) dz \\ &= \int_o^{z_{po}} \left(\int_o^{z} \frac{V(x)}{A(x)}\, dx\right) A(z) dz + \int_{z_{po}}^{z_p} \left(\int_o^{z} \frac{V(x)}{A(x)}\, dx\right) A_B dz \\ &= J_3(z_{po}) + \int_{z_{po}}^{z_p} \left(J_1(z_{po}) + \frac{1}{A_B}\left(V(z_{po})(z - z_{po}) + \frac{A_B}{2}\ (z - z_{po})^2\right)\right) A_B\ dz \\ &= J_3(z_{po}) + A_B J_1(z_{po})\ (z_p - z_{po}) + \frac{V(z_{po})\ (z_p - z_{po})^2}{2} \\ &+ \frac{A_B}{6}\ (z_p - z_{po})^3, \end{aligned} \tag{C.46}$$

$$\begin{aligned} J_4(z_p) &= \int_o^{z_p} \frac{V^2(z)}{A(z)}\, dz = \int_o^{z_{po}} \frac{V^2(z)}{A(z)}\, dz + \int_{z_{po}}^{z_p} \frac{[V(z_{po}) + A_B(z - z_{po})]^2}{A_B}\, dz \\ &= J_4(z_{po}) + \frac{[V(z_{po}) + A_B(z_p - z_{po})]^3 - V^3(z_{po})}{3A_B^2}. \end{aligned} \tag{C.47}$$

Equations (C.44) - (C.47) require the evaluation of the integrals $J_1(z_{po})$ - $J_4(z_{po})$ only once.

The kinetic energy (KE) of the gas/solid mixture will be required, and since $\rho\, dV = dm$ and $dV = A(z)\, dz$, then

$$g_o KE = \frac{1}{2}\int_o^{z_p} u^2 dm = \frac{1}{2}\int_o^{z_p} u^2 \rho dV = \frac{1}{2}\int_o^{z_p} u^2 \rho A(z) dz \qquad (C.48)$$

and from (C.5),

$$\rho = \frac{C}{V(z_p)},$$

and from (C.14)

$$u = \frac{A_B V_p V(z)}{V(z_p) A(z)},$$

therefore,

$$g_o KE = \frac{1}{2}\int_o^{z_p} \frac{C A_B^2 V_p^2 V^2(z)}{A(z) V^3(z_p)}\, dz$$

$$= \frac{C A_B^2 V_p^2}{2V^3(z_p)} \left(\int_o^{z_p} \frac{V^2(z)}{A(z)}\, dz\right),$$

or

$$KE = \frac{C A_B^2 V_p^2}{2 g_o V^3(z_p)}\, J_4(z_p). \qquad (C.49)$$

APPENDIX 2:
INFLUENCE OF PROPELLANT VELOCITY LAG

INTENTIONALLY LEFT BLANK.

The following, is work by Gough (Gough, in preparation) with modifications and elaborations by Robbins.

Influence of Propellant Velocity Lag. For the effects of the propellant velocity lag on the form of the gradient equation, the propellant is assumed to be uniformly distributed between the breech face and the leading edge of the bed, with the leading edge of the bed being initially at the base of the projectile. The tube diameter is the same as the bore diameter.

The continuity equations for each of the phases in the mixture region are

$$\frac{\partial \varepsilon \rho}{\partial t} + \frac{\partial \varepsilon \rho u}{\partial z} = \dot{m}(t). \tag{L.1}$$

$$\rho_p \frac{\partial \varepsilon}{\partial t} - \frac{\partial \; [(1-\varepsilon) \;\; \rho_p u_p]}{\partial z} = \dot{m}(t). \tag{L.2}$$

Making assumptions analogous to the Lagrange assumption,

$$\frac{\partial \rho}{\partial z} = 0 \quad (\textit{gas density constant throughout the tube}) \tag{L.3}$$

$$\frac{\partial \varepsilon}{\partial z} = 0 \quad (\textit{porosity constant throughout the mixture region}) \tag{L.4}$$

and noting the assumption that $\dot{m}(t)$ is only a function of time, then from (L.2),

$$\rho_p \frac{\partial \varepsilon}{\partial t} - u_p \frac{\partial \; (1-\varepsilon) \;\; \rho_p}{\partial z} - (1-\varepsilon) \;\; \rho_p \frac{\partial u_p}{\partial z} = \dot{m}(t). \tag{L.5}$$

Noting (L.4), then

$$u_p \frac{\partial \; (1-\varepsilon) \;\; \rho_p}{\partial z} = 0. \tag{L.6}$$

Therefore,

$$(1-\varepsilon) \;\; \rho_p \frac{\partial u_p}{\partial z} = \rho_p \frac{\partial \varepsilon}{\partial t} - \dot{m}(t) = f(t) \tag{L.7}$$

and

$$u_p(z) \;=\; \int \partial u_p \;=\; \frac{\rho_p \frac{\partial \varepsilon}{\partial t} \;-\; \dot{m}(t)}{(1-\varepsilon) \;\; \rho_p} \; z \;+\; f_1(t). \tag{L.8}$$

Since $u_p(0) = 0$, then $f_1(t) = 0$, and

$$u_p(z_b) \;=\; U_p \;=\; \frac{\rho_p \frac{\partial \varepsilon}{\partial t} \;-\; \dot{m}(t)}{(1-\varepsilon) \;\; \rho_p} \; z_b,$$

or

$$\frac{U_p}{z_b} \;=\; \frac{\rho_p \frac{\partial \varepsilon}{\partial t} \;-\; \dot{m}(t)}{(1-\varepsilon) \;\; \rho_p}. \tag{L.9}$$

Substituting (L.9) into (L.8) gives

$$u_p(z) \;=\; \frac{U_p}{z_b} \; z. \tag{L.10}$$

From (L.1),

$$\frac{\partial \varepsilon\rho}{\partial t} \;+\; u \left(\rho \; \frac{\partial \varepsilon}{\partial z} \;+\; \varepsilon \; \frac{\partial \rho}{\partial z} \right) \;+\; \varepsilon\rho \; \frac{\partial u}{\partial z} \;=\; \dot{m}(t),$$

and using (L.3) and (L.4), then

$$\frac{\partial u}{\partial z} \;=\; \frac{-\; \frac{\partial \varepsilon\rho}{\partial t} \;+\; \dot{m}(t)}{\varepsilon\rho} \quad (\textit{a function of time only}). \tag{L.11}$$

Integrating (L.11) gives

$$u(z) \;=\; \int \partial u \;=\; \frac{-\; \frac{\partial \varepsilon\rho}{\partial t} \;+\; \dot{m}(t)}{\varepsilon\rho} \; z \;+\; f_2(t). \tag{L.12}$$

Since $u(0) = 0$, then $f_2(t) = 0$, and

$$u(z_b) \;=\; U_g \;=\; \frac{-\; \frac{\partial \varepsilon\rho}{\partial t} \;+\; \dot{m}(t)}{\varepsilon\rho} \; z_b$$

or

$$\frac{U_g}{z_b} \;=\; \frac{-\; \frac{\partial \varepsilon\rho}{\partial t} \;+\; \dot{m}(t)}{\varepsilon\rho}. \tag{L.13}$$

Substituting (L.13) into (L.12) gives

$$u(z) = \frac{U_g}{z_b} z. \qquad (L.14)$$

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form

$$\varepsilon \ (U_g - U_p) = U_{g+} - U_p. \qquad (L.15)$$

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption L.3.

Also, in the ullage region, the continuity equation is given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} = 0. \qquad (L.16)$$

Using (L.3), we get

$$\frac{\partial u}{\partial z} = - \frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right) \ (\textit{a function of time only}). \qquad (L.17)$$

Integrating (L.17) we get

$$u(z) = - \frac{1}{\rho} \left(\frac{d\rho}{dt} \right) z + f_3(t). \qquad (L.18)$$

Using the boundary condition

$$u(z_p) = V_p \qquad (L.19)$$

gives

$$u(z) = V_p + (z_p - z) \frac{dln\rho}{dt}. \qquad (L.20)$$

Evaluating (L.20) at z_b gives

$$U_{g+} = V_p + \frac{L}{\rho} \left(\frac{d\rho}{dt} \right) = V_p + L \frac{dln\rho}{dt}. \qquad (L.21)$$

Eliminating U_{g+} from (L.15) with (L.21) gives

$$\varepsilon \ (U_g - U_p) = V_p + L \frac{dln\rho}{dt} - U_p \qquad (L.22)$$

or

$$U_g = U_p + \frac{1}{\varepsilon}\left(V_p + L\frac{dln\rho}{dt} - U_p\right). \qquad (L.23)$$

Differentiating (L.23) yields

$$\dot{U}_g = -\dot{U}_p\left(\frac{1-\varepsilon}{\varepsilon}\right) - \frac{\dot{\varepsilon}}{\varepsilon^2}\left(V_p + L\frac{dln\rho}{dt} - U_p\right)$$

$$+ \frac{\dot{V}_p}{\varepsilon} + \frac{L}{\varepsilon}\left(\frac{d^2\ln\rho}{dt^2}\right) + \frac{\dot{L}}{\varepsilon}\left(\frac{dln\rho}{dt}\right). \qquad (L.24)$$

We anticipate the need to factor out the base pressure P_B dependence of both $\dot{V}_p$ and $d^2\ln\rho/dt^2$.

The projectile acceleration is given by

$$\dot{V}_p = \frac{g_o A_B P_B}{M_p} - \frac{g_o A_B P_{res}}{M_p}. \qquad (L.25)$$

To get the base pressure P_B dependence associated with $d^2\ln\rho/dt^2$, we start with the definition of the gas density

$$\rho = \frac{m}{V_F} \qquad (L.26)$$

where m = mass of gas burned and V_F = free volume. Therefore,

$$V_F = V_o + A_B\ (z_p - z_{po}) - \frac{C}{\rho_p} + \frac{m}{\rho_p} \qquad (L.27)$$

and

$$\dot{V}_F = A_B\dot{z}_p + \frac{\dot{m}}{\rho_p} = A_B V_p + \frac{\dot{m}}{\rho_p} \qquad (L.28)$$

and

$$\ddot{V}_F = A_B\dot{V}_p + \frac{\ddot{m}}{\rho_p} \qquad (L.29)$$

where it is understood that $m/\rho_p, \dot{m}/\rho_p$, and $\ddot{m}/\rho_p$ may be in the form $\Sigma\frac{m_i}{\rho_{p_i}}$, $\Sigma\frac{\dot{m}_i}{\rho_{p_i}}$, $\Sigma\frac{\ddot{m}_i}{\rho_{p_i}}$, where the i index refers to different propellants.

From (L.26) we get

$$\ln\rho = \ln m - \ln V_F, \tag{L.30}$$

and

$$\frac{d\ln\rho}{dt} = \frac{\dot{m}}{m} - \frac{\dot{V}_F}{V_F} \tag{L.31}$$

and

$$\frac{d^2\ln\rho}{dt^2} = \frac{\ddot{m}}{m} - \frac{\dot{m}^2}{m^2} - \frac{\ddot{V}_F}{V_F} + \frac{\dot{V}_F^2}{V_F^2}. \tag{L.32}$$

Substituting (L.29) and (L.25), we get

$$\frac{d^2\ln\rho}{dt^2} = c_1(t) - \frac{g_o A_B^2 P_B}{V_F M_p} + \frac{g_o A_B^2 P_{res}}{V_F M_p}, \tag{L.33}$$

where

$$c_1(t) = \ddot{m}\left(\frac{1}{m} - \frac{1}{\rho_p V_F}\right) + \frac{\dot{V}_F^2}{V_F^2} - \frac{\dot{m}^2}{m^2}.$$

We still require $\ddot{m}$, which we get from

$$\dot{m} = \rho_p \; S \; \frac{dx}{dt} \tag{L.34}$$

where S is the surface area of the propellant and dx/dt is the burning rate of the propellant, where dx/dt is of the form

$$\frac{dx}{dt} = aP_m^n. \tag{L.35}$$

Therefore, from (L.34),

$$\begin{aligned}\ddot{m} &= \rho_p \; \dot{S} \; \frac{dx}{dt} + \rho_p \; S \; \frac{d^2x}{dt^2} \\ &= \rho_p \; \frac{dS}{dx}\left(\frac{dx}{dt}\right)^2 + \rho_p \; S \; \frac{d^2x}{dt^2},\end{aligned} \tag{L.36}$$

and from (L.35),

$$\frac{d^2x}{dt^2} = anP_m^{n-1}\;\frac{dP_m}{dt} \tag{L.37}$$

where dP_m/dt is determined numerically or from

$$\frac{dP_m}{dt} = \frac{\frac{\dot{m}RT}{m_w} - P_m\dot{V} + \frac{mR\dot{T}}{m_w}}{V}.$$

Also defining ϕ as the mass fraction of propellant burned, then

$$\rho = \frac{\phi C - \rho A_B L}{\varepsilon V(z_b)} = \frac{\phi_* C}{\varepsilon V(z_b)} \tag{L.38}$$

where

$$\phi_* = \phi - \frac{\rho A_B L}{C} \tag{L.39}$$

and

$$\rho_p = \frac{(1-\phi)C}{(1-\varepsilon)V(z_b)}. \tag{L.40}$$

Differentiating (L.39), we get

$$\dot{\phi}_* = \dot{\phi} - \frac{\rho A_B}{C}\left(\dot{L} + L\,\frac{d\ln\rho}{dt}\right). \tag{L.41}$$

Solving (L.40) for ε, we get

$$\varepsilon = 1 - \frac{(1-\phi)C}{\rho_p V(z_b)}. \tag{L.42}$$

Differentiating (L.42) and noting that

$$V(z_b) = V(z_{po}) + (z_b - z_{po})\ A_B \tag{L.43}$$

and, therefore,

$$\dot{V}(z_b) = \dot{z}_b A_B = U_p A_B, \tag{L.44}$$

then

$$\dot{\varepsilon} = \frac{\dot{\phi} C}{\rho_p V(z_b)} + \frac{(1-\phi)\ C\dot{V}(z_b)}{\rho_p V^2(z_b)} = \frac{\dot{\phi} C}{\rho_p V(z_b)} + \frac{(1-\phi)\ C U_p A_B}{\rho_p V^2(z_b)}. \tag{L.45}$$

We now have the quantities which are required for the solution of the momentum equation from which we get the pressure distribution in the mixture region.

Consider the momentum equations for each phase

$$\frac{\partial \varepsilon \rho u}{\partial t} + \frac{\partial \varepsilon \rho u^2}{\partial z} + \varepsilon g_o \frac{\partial P}{\partial z} = - f_s + \dot{m} u_p \tag{L.46}$$

$$\frac{\partial \ (1-\varepsilon) \ \rho_p u_p}{\partial t} + \frac{\partial \ (1-\varepsilon) \ \rho_p u_p^2}{\partial z} + (1-\varepsilon) \ g_o \frac{\partial P}{\partial z} = f_s - \dot{m} u_p \tag{L.47}$$

where

$$f_s = \frac{(1-\varepsilon)}{D_p} \rho \ (u-u_p)^2 \ f_{sc} \tag{L.48}$$

and

$$f_{sc} = \begin{cases} \dfrac{2.5\lambda}{REN^{.081}} \ f_{so} & \varepsilon < \varepsilon_o \\ \\ \dfrac{2.5\lambda}{REN^{.081}} \left(\dfrac{(1-\varepsilon)}{(1-\varepsilon_o)} \ \dfrac{\varepsilon_o}{\varepsilon} \right)^{0.45} f_{so}, & \varepsilon \geq \varepsilon_o \end{cases} \tag{L.49}$$

where

$$\lambda = \left(\frac{0.5 + \frac{GL}{GD}}{\left(1.5 \ \frac{GL}{GD}\right)^{2/3}} \right)^{2.17} \tag{L.50}$$

and

$$REN = \rho D_p \, | U_g - U_p | / \mu .$$

Adding (L.46) and (L.47), we get

$$g_o \frac{\partial P}{\partial z} = - \frac{\partial \varepsilon \rho u}{\partial t} - \frac{\partial \ (1-\varepsilon) \ \rho_p u_p}{\partial t} - \frac{\partial \ \varepsilon \rho u^2}{\partial z} - \frac{\partial \ (1-\varepsilon) \ \rho_p u_p^2}{\partial z} \tag{L.51}$$

or

$$g_o \frac{\partial P}{\partial z} = - \frac{\partial}{\partial t} [\varepsilon \rho u + (1-\varepsilon) \ \rho_p u_p] - \frac{\partial}{\partial z} [\varepsilon \rho u^2 + (1-\varepsilon) \ \rho_p u_p^2]. \tag{L.52}$$

From (L.10) we get $u_p(z) = U_p[z/z_b]$, from (L.14) we get $u(z) = U_g[z/z_b]$, from (L.38) we get

$$\varepsilon\rho = \frac{\phi C - \rho A_B L}{V(z_b)} = \frac{\phi_* C}{V(z_b)},$$

and from (L.40) we get

$$(1-\varepsilon)\rho_p = \frac{(1-\phi)C}{V(z_b)},$$

which, when substituted into (L.52), yields

$$g_o \frac{\partial P}{\partial z} = -\frac{\partial}{\partial t}\left(\frac{\phi_* C U_g z}{V(z_b) z_b} + \frac{(1-\phi)\ C U_p z}{V(z_b) z_b}\right) - \frac{\partial}{\partial z}\left(\frac{\phi_* C U_g^2 z^2}{V(z_b) z_b^2} + \frac{(1-\phi)\ C U_p^2 z^2}{V(z_b) z_b^2}\right). \qquad (L.53)$$

Differentiating (L.53) and noting z_b, $V(z_b)$, ϕ_*, ϕ, U_g, and U_p are functions of time only and that $\dot{z}_b = U_p$, $\dot{V}(z_b) = A_B U_p$, and $V(z_b) = A_B z_b$, we get

$$\begin{aligned} g_o \frac{\partial P}{\partial z} = &-zC\left(\frac{\phi_* \dot{U}_g + \dot{\phi}_* U_g + (1-\phi)\ \dot{U}_p - \dot{\phi} U_p}{z_b V(z_b)}\right) \\ &+ \frac{zC\ [\phi_* U_g + (1-\phi)\ U_p]\ \dot{z}_b}{z_b^2 V(z_b)} \\ &+ \frac{zC\ [\phi_* U_g + (1-\phi)\ U_p]\ \dot{V}(z_b)}{z_b V^2(z_b)} \\ &- \frac{2zC\ [\phi_* U_g^2 + (1-\phi) U_p^2]}{z_b^2 V(z_b)} \end{aligned} \qquad (L.54)$$

or

$$\begin{aligned} g_o \frac{\partial P}{\partial z} = -\frac{zC}{z_b V(z_b)}\Bigg(&\phi_* \dot{U}_g + \dot{\phi}_* U_g + (1-\phi)\dot{U}_p - \dot{\phi} U_p \\ &- \frac{[\phi_* U_g + (1-\phi) U_p] U_p}{z_b} - \frac{[\phi_* U_g + (1-\phi) U_p] U_p}{z_b} \\ &+ 2\ \frac{[\phi_* U_g^2 + (1-\phi) U_p^2]}{z_b}\Bigg) \end{aligned} \qquad (L.55)$$

or

$$g_o \frac{\partial P}{\partial z} = - \frac{zC}{z_b V(z_b)} \left(\phi_* \dot{U}_g + \dot{\phi}_* U_g + (1-\phi) \dot{U}_p - \dot{\phi} U_p + \frac{2\phi_* U_g (U_g - U_p)}{z_b} \right). \quad (L.56)$$

To get $\dot{U}_p$ for substitution into (L.56), we use another form of the solid phase momentum equation

$$\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z} + \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} = \frac{f_s}{(1-\varepsilon)\rho_p} , \quad (L.57)$$

and noting that at $z = z_b$, $\partial u_p(z_b)/\partial t = \dot{U}_p$, and that $\partial U_p/\partial z = 0$ (since U_p is a function of time only), then

$$\dot{U}_p + \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} |z_b = \frac{f_s}{(1-\varepsilon) \ \rho_p}. \quad (L.58)$$

Substituting (L.48) for f_s in (L.58), we get

$$\dot{U}_p = \frac{\rho \ (U_g - U_p)^2}{D_p \rho_p} f_{sc} - \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} | z_b. \quad (L.59)$$

Substituting (L.24) into (L.56), we get

$$\begin{aligned} g_o \frac{\partial P}{\partial z} = - \frac{zC}{z_b V(z_b)} \Bigg(& \phi_* \frac{\dot{V}_p}{\varepsilon} - \frac{(1-\varepsilon)}{\varepsilon} \dot{U}_p \ \phi_* \\ & - \frac{\dot{\varepsilon}}{\varepsilon^2} \phi_* \left(V_p + L \frac{dln\rho}{dt} - U_p \right) + \frac{L\phi_*}{\varepsilon} \frac{d^2 \ln\rho}{dt^2} \\ & + \frac{\phi_* \dot{L}}{\varepsilon} \frac{dln\rho}{dt} + \dot{\phi}_* U_g + (1-\phi) \ \dot{U}_p - \dot{\phi} U_p \\ & + \frac{2\phi_* U_g}{z_b} (U_g - U_p) \Bigg). \end{aligned} \quad (L.60)$$

Substituting (L.58) into (L.60), we get

$$g_o \frac{\partial P}{\partial z} = - \frac{zC}{z_b V(z_b)} \left(\phi_* \frac{\dot{V}_p}{\varepsilon} + \dot{\phi}_* U_g - \dot{\phi} U_p \right.$$

$$- \frac{\dot{\varepsilon}\phi_*}{\varepsilon^2} \left(V_p + L \frac{d\ln\rho}{dt} - U_p \right) + \frac{\phi_* L}{\varepsilon} \frac{d^2 \ln\rho}{dt^2}$$

$$+ \frac{\phi_* \dot{L}}{\varepsilon} \frac{d\ln\rho}{dt} + \frac{2\phi_* U_g}{z_b} (U_g - U_p) + \frac{\phi_2 \rho}{D_p \rho_p} (U_g - U_p)^2 f_{sc}$$

$$\left. - \frac{\phi_2 g_o}{\rho_p} \frac{\partial P}{\partial z} |z_b \right) \qquad (L.61)$$

or

$$g_o \frac{\partial P}{\partial z} = - \frac{zC}{z_b V(z_b)} \left(\phi_1 - \frac{\phi_2 g_o}{\rho_p} \frac{\partial P}{\partial z} |z_b \right) \qquad (L.62)$$

where

$$\phi_1 = \phi_1' + \frac{\phi_* \dot{V}p}{\varepsilon} + \frac{L\phi_*}{\varepsilon} \frac{d^2 \ln\rho}{dt^2} \qquad (L.63)$$

and

$$\phi_1' = \dot{\phi}_* U_g - \dot{\phi} U_p - \frac{\phi_* \dot{\varepsilon}}{\varepsilon^2} \left(V_p + L \frac{d\ln\rho}{dt} - U_p \right)$$

$$+ \frac{\phi_* \dot{L}}{\varepsilon} \frac{d\ln\rho}{dt} + \frac{2\phi_* U_g}{z_b} (U_g - U_p) + \frac{\phi_2 \rho}{D_p \rho_p} (U_g - U_p)^2 f_{sc} \qquad (L.64a)$$

and

$$\phi_2 = 1 - \phi - \phi_* \frac{(1-\varepsilon)}{\varepsilon}. \qquad (L.64b)$$

To get $\frac{\partial P}{\partial z}|z_b$ for elimination from (L.62), we evaluate (L.62) at z_b and solve for $\frac{\partial P}{\partial z}|z_b$. (L.62) evaluated at z_b is

$$g_o \frac{\partial P}{\partial z}|z_b = - \frac{z_b C}{z_b V(z_b)} \left(\phi_1 - \phi_2 \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} |z_b \right) \qquad (L.65)$$

or

$$\frac{\partial P}{\partial z} |_{z_b} = - \frac{C\phi_1}{g_o V(z_b) \left(1 - \frac{\phi_2 C}{\rho_p V(z_b)} \right)}. \qquad (L.66)$$

Substituting (L.66) back into (L.62), we get

$$g_o \frac{\partial P}{\partial z} = - \frac{zC}{z_b V(z_b)} \left(\phi_1 + \frac{\phi_2 g_o C \phi_1}{\rho_p g_o V(z_b) \left(1 - \frac{\phi_2 C}{\rho_p V(z_b)}\right)} \right) \tag{L.67}$$

or

$$\frac{\partial P}{\partial z} = - \frac{zC}{g_o z_b V(z_b)} \left(\frac{\phi_1}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right) \tag{L.68}$$

or

$$\frac{\partial P}{\partial z} = - k_1(t) z \tag{L.69}$$

where

$$k_1(t) = k_1 = \frac{C}{g_o z_b V(z_b)} \left(\frac{\phi_1}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right). \tag{L.70}$$

k_1 depends upon the base pressure $P_B = P(z_p)$ (one of the values we wish to solve for) through $\dot{V}_p$ (the acceleration of the projectile) and also through $(d^2 \ln \rho)/(dt^2)$.

Substituting (L.25),

$$\dot{V}_p = \frac{g_o A_B P_B}{M_p} - \frac{g_o A_B P_{res}}{M_p}$$

and (L.33) into (L.70), we get

$$k_1 = \frac{C}{g_o z_b V(z_b)} \frac{\phi_1' + D - EP_{res} + EP_B}{\left(1 - \frac{\phi_2 C}{\rho_p V(z_b)}\right)} \tag{L.71}$$

where

$$D = \frac{L \phi_* c_1(t)}{\varepsilon} \tag{L.72}$$

and

$$E = \frac{\phi_*}{\varepsilon} \left(1 - \frac{L A_B}{V_F} \right) \frac{g_o A_B}{M_p} \tag{L.73}$$

or

$$k_1 = k_{11}P_B + k_{12} \tag{L.74}$$

where

$$k_{11} = \frac{CEk_2}{g_o z_b V(z_b)} \tag{L.75}$$

$$k_2 = \frac{1}{1-\frac{\phi_2 C}{\rho_p V(z_b)}} \tag{L.76}$$

and

$$k_{12} = \frac{(\phi_1' + D)\ Ck_2}{g_o z_b V(z_b)} - k_{11}\ P_{res}. \tag{L.77}$$

Therefore, (L.69) becomes

$$\frac{\partial P}{\partial z} = -\ (k_{11}P_B + k_{12})\ z. \tag{L.78}$$

Integrating (L.78) gives us the pressure distribution in the mixture $0 < z < z_b$

$$P(z) = P(0) - \frac{k_{11}P_B + k_{12}}{2}\ z^2. \tag{L.79}$$

We now need the pressure distribution in the ullage region. For the ullage region, the momentum equation for the gas is

$$\frac{\partial P}{\partial z} = -\ \frac{\rho}{g_o}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z}\right). \tag{L.80}$$

The equation of continuity in the ullage region is

$$\frac{\partial u}{\partial z} = -\ \frac{1}{\rho}\ \frac{d\rho}{dt} = -\ \frac{d\ln\rho}{dt}. \tag{L.81}$$

Integrating (L.81) and noting $(d\ln\rho)/dt$ is a function of time only, we get

$$u(z) = -\ \frac{d\ln\rho}{dt}\ z + f(t) \tag{L.82}$$

with the boundary condition,

$$u(z_p) = V_p = -z_p \frac{dln\rho}{dt} + f(t) \qquad (L.83)$$

Therefore

$$f(t) = V_p + z_p \frac{dln\rho}{dt} \qquad (L.84)$$

and

$$u(z) = -\frac{dln\rho}{dt} z + V_p + z_p \frac{dln\rho}{dt} = V_p + (z_p - z) \frac{dln\rho}{dt}. \qquad (L.85)$$

Differentiating (L.85) gives

$$\frac{\partial u}{\partial t} = \dot{V}_p + (z_p - z) \frac{d^2 \ln\rho}{dt^2} + \dot{z}_p \frac{dln\rho}{dt} \qquad (L.86)$$

or

$$\frac{\partial u}{\partial t} = \dot{V}_p + (z_p - z) \frac{d^2 \ln\rho}{dt^2} + V_p \frac{dln\rho}{dt}. \qquad (L.87)$$

Substituting (L.81), (L.85), and (L.87) into (L.80), we get

$$\frac{\partial P}{\partial z} = -\frac{\rho}{g_o} \left(\dot{V}_p + (z_p - z) \frac{d^2 \ln\rho}{dt^2} + V_p \frac{dln\rho}{dt} \right.$$
$$\left. - \left(V_p + (z_p - z) \frac{dln\rho}{dt} \right) \left(\frac{dln\rho}{dt} \right) \right) \qquad (L.88)$$

or

$$\frac{\partial P}{\partial z} = -\frac{\rho}{g_o} \left(\dot{V}_p + (z_p - z) \frac{d^2 \ln\rho}{dt^2} - (z_p - z) \left(\frac{dln\rho}{dt} \right)^2 \right). \qquad (L.89)$$

Integrating (L.89) from z_b to z and noting z_p, $\dot{V}_p$, and $\ln\rho$ are functions of time only, we get

$$P(z) = P(z_b) - \frac{\rho}{g_o} \left(\dot{V}_p z - \frac{(z_p - z)^2}{2} \left(\frac{d^2 \ln\rho}{dt^2} \right) \right.$$
$$\left. + \frac{(z_p - z)^2}{2} \left(\frac{dln\rho}{dt} \right)^2 \right) \Big|_{z_b}^{z} \qquad (L.90)$$

$$P(z) = P(z_b) - \frac{\rho}{g_o}\left(\dot{V}_p z - \frac{(z_p - z)^2}{2}\Delta\right)\Big|_{z_b}^{z} \qquad (L.91)$$

where

$$\Delta = \frac{d^2 \ln\rho}{dt^2} - \left(\frac{d\ln\rho}{dt}\right)^2. \qquad (L.92)$$

Therefore, we get the pressure distribution in the ullage region as

$$P(z) = P(z_b) - \frac{\rho}{g_o}\left(\dot{V}_p(z - z_b) + [(z_p - z_b)^2 - (z_p - z)^2]\frac{\Delta}{2}\right) \qquad (L.93)$$

or

$$P(z) = P(z_b) - \frac{\rho}{g_o}\left((\dot{V}_p + z_p\Delta)(z - z_b) - (z^2 - z_b^2)\frac{\Delta}{2}\right). \qquad (L.94)$$

Defining the mean pressure P_m to be

$$P_m = \frac{\int_o^{z_p} P(z)A(z)dz}{\int_o^{z_p} A(z)dz} \qquad (L.95)$$

and, since $A(z) = A_B$ is a constant, then

$$P_m = \frac{\int_o^{z_b} P(z)dz + \int_{z_b}^{z_p} P(z)dz}{z_p}. \qquad (L.96)$$

Substituting the pressure distribtution in the mixture region (L.79) and the pressure distribution in the ullage region (L.94) into (L.96) to get the mean pressure, and noting that $P(0) = P_{Br}$ (the pressure at the breech) we get

$$P_m = \frac{1}{z_p}\left(\int_o^{z_b}\left(P_{Br} - \frac{(k_{11}P_B + K_{12})}{2}z^2\right)dz\right.$$

$$\left. + \int_{z_b}^{z_p}\left(P(z_b) - \frac{\rho}{g_o}\left[(\dot{V}_p + z_p\Delta)(z - z_o) - (z^2 - z_b^2)\frac{\Delta}{2}\right]\right)dz\right) \qquad (L.97)$$

or

$$P_m = \left[P_{Br} z_b - \frac{(k_{11}P_B + k_{12})}{6} z_b^3 + P(z_b) \quad (z_p - z_b) - \frac{\rho}{2g_o}\left((\dot{V}_p + z_p\Delta) \quad (z_p - z_b)^2 - \frac{\Delta}{3} (z_p^3 - z_b^3) + z_b^2\Delta \quad (z_p - z_b)\right)\right] / z_p \tag{L.98}$$

or, since $L = z_p - z_b$,

$$P_m = P_{Br}\frac{z_b}{z_p} - \frac{k_{11}P_b z_b^3}{6z_p} - \frac{k_{12}z_b^3}{6z_p} + \frac{P(z_b)L}{z_p} - \frac{\rho L^2}{2g_o z_p}\left(\dot{V}_p + \frac{2}{3} \Delta L\right). \tag{L.99}$$

We need to substitute for $P(z_b)$. To get $P(z_b)$, we evaluate (L.79) at z_b and get

$$P(z_b) = P_{Br} - \frac{k_{11}z_b^2 P_b}{2} - \frac{k_{12}z_b^2}{2}. \tag{L.100}$$

Substituting (L.100) into (L.99), we get

$$P_m = P_{Br}\frac{z_b}{z_p} - \frac{k_{11}z_b^3 P_b}{6z_p} - \frac{k_{12}z_b^3}{6z_p} + \frac{L}{z_p}\left(P_{Br} - \frac{k_{11}z_b^2 P_b}{2} - \frac{k_{12}z_b^2}{2}\right) - \frac{\rho L^2 \dot{V}_p}{2g_o z_p} - \frac{\rho L^3 \Delta}{3g_o z_p}. \tag{L.101}$$

Substituting (L.25) for $\dot{V}_p$ and combining terms, we get

$$P_m = P_{Br} + P_B\left(-\frac{k_{11}z_b^3}{6z_p} - \frac{Lk_{11}z_b^2}{2z_p} - \frac{\rho L^2 A_B}{2z_p M_p}\right) + \frac{\rho L^2 A_B P_{res}}{2z_p M_p} - \frac{k_{12}z_b^3}{6z_p} - \frac{Lk_{12}z_b^2}{2z_p} - \frac{\rho L^3 \Delta}{3g_o z_p}. \tag{L.102}$$

Evaluating (L.94) at z_p, we get

$$P(z_p) = P(z_b) - \frac{\rho}{g_o}\left[(\dot{V}_p + z_p\Delta) \quad (z_p - z_b) - (z_p^2 - z_b^2) \quad \frac{\Delta}{2}\right], \tag{L.103}$$

and substituting (L.100) into (L.103) and noting $L = z_p - z_b$, we get

$$P(z_p) = P_B = P_{Br} - \frac{k_{11} z_b^2 P_b}{2} - \frac{k_{12} z_b^2}{2} - \frac{\rho L \dot{V}_p}{g_o} - \frac{\rho L z_p \Delta}{g_o}$$

$$+ \frac{\rho}{g_o} \left(z_p^2 - z_b^2\right) \frac{\Delta}{2}. \qquad (L.104)$$

Substituting (L.25) for $\dot{V}_p$, and collecting P_B terms for (L.104), we get

$$0 = P_{Br} + P_B \left(-1 - \frac{k_{11} z_b^2}{2} - \frac{\rho L A_B}{M_p}\right) - \frac{k_{12} z_b^2}{2} + \frac{\rho L A_B P_{res}}{M_p}$$

$$- \frac{\rho L z_p \Delta}{g_o} + \frac{\rho \Delta}{2 g_o} \left(z_p^2 - z_b^2\right) \qquad (L.105)$$

or

$$0 = P_{Br} + P_B \left(-1 - \frac{k_{11} z_b^2}{2} - \frac{\rho L A_B}{M_p}\right) - \frac{k_{12} z_b^2}{2} + \frac{\rho L A_B P_{res}}{M_p}$$

$$- \frac{\rho L^2 \Delta}{2 g_o}. \qquad (L.106)$$

Subtracting (L.106) from (L.102) to eliminate P_{Br} and get P_B in terms of P_m, we get

$$P_m = P_B \left(- \frac{k_{11} z_b^3}{6 z_p} - \frac{L k_{11} z_b^2}{2 z_p} - \frac{\rho L^2 A_B}{2 z_p M_p} + 1 + \frac{k_{11} z_b^2}{2} + \frac{\rho L A_B}{M_p}\right)$$

$$+ \frac{\rho L^2 A_B P_{res}}{2 z_p M_p} - \frac{k_{12} z_b^3}{6 z_p} - \frac{L k_{12} z_b^2}{2 z_p} + \frac{k_{12} z_b^2}{2}$$

$$- \frac{\rho L A_B P_{res}}{M_p} + \frac{\rho L^2 \Delta}{2 g_o} \left(1 - \frac{2L}{3 z_p}\right). \qquad (L.107)$$

Noting the dependence of P_B in Δ, we use (L.92)

$$\Delta = \frac{d^2 \ln \rho}{dt^2} - \left(\frac{d \ln \rho}{dt}\right)^2$$

or, substituting for

$$\frac{d^2 \ln \rho}{dt^2}$$

from (L.33) and elminating $\dot{V}_p$ with (L.25), we get

$$\Delta = c_1(t) - \frac{g_o A_B^2 P_B}{V_F M_p} + \frac{g_o A_B P_{res}}{M_p V_F} - \left(\frac{d\ln\rho}{dt}\right)^2. \qquad (L.108)$$

Substituting (L.108) into (L.107) and collecting P_B terms, we get

$$\begin{aligned} P_m = P_B \Bigg[& - \frac{k_{11} z_b^3}{6 z_p} - \frac{L k_{11} z_b^2}{2 z_p} - \frac{\rho L^2 A_B}{2 z_p M_p} + 1 + \frac{k_{11} z_b^2}{2} + \frac{\rho L A_B}{M_p} \\ & - \frac{A_B^2 \rho L^2}{2 V_F M_p} \left(1 - \frac{2L}{3 z_p}\right)\Bigg] + \frac{k_{12} z_b^2}{2} - \frac{\rho L A_B P_{res}}{M_p} - \frac{k_{12} z_b^2}{2 z_p} \left(\frac{z_b}{3} + L\right) \\ & + \frac{\rho L^2 A_B P_{res}}{2 z_p M_p} + \frac{\rho L^2}{2 g_o} \left(1 - \frac{2L}{3 z_p}\right) \left(c_1(t) - \left(\frac{d\ln\rho}{dt}\right)^2\right) \\ & + \frac{A_B^2 \rho L^2}{2 M_p V_F} \left(1 - \frac{2L}{3 z_p}\right) P_{res}. \end{aligned} \qquad (L.109)$$

Substituting (L.108) into (L.106) and collecting P_B terms, we get

$$\begin{aligned} P_{Br} = P_B & \left(1 + \frac{k_{11} z_b^2}{2} + \frac{\rho L A_B}{M_p} - \frac{A_B^2 \rho L^2}{2 V_F M_p}\right) + \frac{k_{12} z_b^2}{2} - \frac{\rho L A_B P_{res}}{M_p} \\ & + \frac{\rho L^2}{2 g_o} \left(c_1(t) - \left(\frac{d\ln\rho}{dt}\right)^2\right) + \frac{A_B^2 \rho L^2}{2 V_F M_p} P_{res}. \end{aligned} \qquad (L.110)$$

Equation (L.109) gives P_B in terms of P_m (which is determined using an equation of state) and then (L.110) gives P_{Br} in terms of P_B.

The kinetic energy (KE) of the gas and solid will be required. The kinetic energy is given by

$$g_o \; KE = \frac{1}{2} \int_o^{z_p} u^2 dm \qquad (L.111)$$

where u is the velocity and dm is the differential of mass; and since ρdV = dm and dV = A(z) dz = $A_B$dz, then

$$g_o \quad KE = \frac{1}{2} \int_o^{z_b} u^2 dm + \frac{1}{2} \int_{z_b}^{z_p} u^2 dm$$

or

$$g_o \quad KE = \frac{1}{2} (1-\varepsilon) \int_o^{z_b} u_p^2 \rho_p A_B dz + \frac{1}{2}\varepsilon \int_o^{z_b} u^2 \rho A_B dz$$

$$+ \frac{1}{2} \int_{z_b}^{z_p} u^2 \rho A_B dz. \qquad (L.112)$$

Substituting (L.10) for u_p, and (L.14) for u in the mixture region, and (L.20) for u in the ullage region, we get

$$g_o \quad KE = \frac{1}{2} (1-\varepsilon) \; \rho_p A_B \frac{U_p^2}{z_b^2} \int_o^{z_b} z^2 dz + \frac{\varepsilon}{2} \frac{\rho A_B U_g^2}{z_b^2} \int_o^{z_b} z^2 dz$$

$$+ \frac{\rho A_B}{2} \int_{z_b}^{z_p} \left(V_p + (z_p - z) \frac{d\ln\rho}{dt} \right)^2 dz, \qquad (L.113)$$

or

$$g_o \quad KE = \frac{A_B z_b}{6} \left[\varepsilon \rho U_g^2 + (1-\varepsilon) \; \rho_p U_p^2 \right]$$

$$- \frac{\rho A_B}{6 \frac{d\ln\rho}{dt}} \left(V_p + (z_p - z) \frac{d\ln\rho}{dt} \right)^3 \Big|_{z_b}^{z_p} \qquad (L.114)$$

or

$$KE = \frac{A_B z_b}{6 g_o} \left[\varepsilon \rho U_g^2 + (1-\varepsilon) \rho_p U_p^2 \right]$$

$$+ \frac{\rho A_B L}{6 g_o} \left[3V_p^2 + 3V_p L \frac{d\ln\rho}{dt} + L^2 \left(\frac{d\ln\rho}{dt} \right)^2 \right]. \qquad (L.115)$$

It would appear that the pressure used in evaluating the burning rate should be the mean pressure over the region occupied by the propellant.

The mean pressure over the mixture region is

$$P_{mix} = \frac{\int_o^{z_b} P(z)A(z)dz}{\int_o^{z_b} A(z)dz}, \qquad (L.116)$$

and since $A(z) = A_B$ and using the pressure distribution in the mixture region given by (L.79), then

$$P_{mix} = \frac{A_B \int_o^{z_b} \left(P_{Br} - \left(\frac{k_{11}P_B + k_{12}}{2}\right)z^2\right)dz}{A_B \int_o^{z_b} dz} \qquad (L.117)$$

or

$$P_{mix} = P_{Br} - \frac{(k_{11}P_B + k_{12})}{6} z_b^2. \qquad (L.118)$$

INTENTIONALLY LEFT BLANK.

APPENDIX 3:
COMBINED INFLUENCE OF CHAMBRAGE AND PROPELLANT VELOCITY LAG

INTENTIONALLY LEFT BLANK.

The following, is work by Gough (Gough, in preparation) with modifications and elaborations by Robbins.

For the effects of the propellant velocity lag and chambrage on the form of the gradient equation, the variation in area is assumed confined to the chamber of the gun and the propellant initially uniformly fills the chamber.

The continuity equations for each of the phases in the mixture region are

$$\frac{\partial \varepsilon \rho}{\partial t} + \frac{1}{A}\frac{\partial \varepsilon \rho u A}{\partial z} = \dot{m}(t) \qquad (R.1)$$

$$\rho_p \frac{\partial \varepsilon}{\partial t} - \frac{1}{A}\frac{\partial \; (1-\varepsilon) \; \rho_p u_p A}{\partial z} = \dot{m}(t). \qquad (R.2)$$

Making assumptions analogous to the Lagrange assumption,

$$\frac{\partial \rho}{\partial z} = 0 \quad (\textit{gas density constant throughout the tube}) \qquad (R.3)$$

$$\frac{\partial \varepsilon}{\partial z} = 0 \quad (\textit{porosity constant throughout the mixture region}) \qquad (R.4)$$

and noting the assumption that $\dot{m}(t)$ is a function of time, then (R.1) becomes,

$$\frac{\partial \varepsilon \rho}{\partial t} + \frac{\varepsilon \rho}{A}\frac{\partial u A}{\partial z} = \dot{m}(t), \qquad (R.4a)$$

or

$$\frac{\partial u A}{\partial z} = \frac{\left(\dot{m}(t) - \frac{\partial \varepsilon \rho}{\partial t}\right) A}{\varepsilon \rho}. \qquad (R.5)$$

Integrating R.5 and noting that the right side is a function of time only except for A which is a function of z only, we get

$$uA = f(t)\int A \, dz + k(t) = f(t) \;\; V(z) + k(t). \qquad (R.6)$$

Applying the boundary condition u(0) = 0 implies that k(t) = 0 since V(0) = 0.

Also $u(z_b) = U_g$ implies that

$$f(t) = \frac{U_g A(z_b)}{V(z_b)}, \tag{R.7}$$

and, since $A(z_b) = A_B$,

$$u(z) = \frac{U_g A_B V(z)}{A(z) V(z_b)}. \tag{R.8}$$

Also (R.2) becomes after using (R.3) and (R.4)

$$\rho_p \frac{\partial \varepsilon}{\partial t} - \frac{(1-\varepsilon)\rho_p}{A} \frac{\partial u_p A}{\partial z} = \dot{m}(t) \tag{R.9}$$

or

$$\frac{\partial u_p A}{\partial z} = \frac{\dot{m}(t) - \rho_p \frac{\partial \varepsilon}{\partial t}}{(1-\varepsilon)\rho_p} A. \tag{R.10}$$

Since the right side of (R.10) is a function of time only except for A, which is a function of z only, on integrating, we get

$$u_p A = f_1(t) \int A \, dz + k_1(t) = f_1(t) V(z) + k_1(t). \tag{R.11}$$

Applying the boundary condition, $u_p(0) = 0$ implies that $k_1(t) = 0$ since $V(0) = 0$.

Also $u_p(z_b) = U_p$ implies that

$$f_1(t) = \frac{U_p A(z_b)}{V(z_b)}, \tag{R.12}$$

and since $A(z_b) = A_B$,

$$u_p(z) = \frac{U_p A_B V(z)}{A(z) V(z_b)}. \tag{R.13}$$

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form

$$\varepsilon \, (U_g - U_p) = U_{g+} - U_p \tag{R.14}$$

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption R.3.

As with the velocity lag derivation, the ullage region is the same.

That is the continuity equation given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} = 0, \qquad (R.15)$$

and using (R.3),

$$\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{d\rho}{dt} \qquad (R.16)$$

where the right side of (R.16) is a function of time only.

Therefore,

$$u = -\frac{1}{\rho}\frac{d\rho}{dt}z + f_3(t). \qquad (R.17)$$

Since $u(z_b) = U_{g+}$, then

$$U_{g+} = -\frac{1}{\rho}\frac{d\rho}{dt}z_b + f_3(t), \qquad (R.18)$$

and since $u(z_p) = V_p$, then

$$V_p = -\frac{1}{\rho}\frac{d\rho}{dt}z_p + f_3(t). \qquad (R.19)$$

Subtracting (R.18) from (R.19) and noting

$$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{d\ln\rho}{dt}$$

we get

$$U_{g+} = \frac{d\ln\rho}{dt}(z_p - z_b) + V_p. \qquad (R.20)$$

Also solving (R.19) for $f_3(t)$ and substituting into (R.17), we get

$$u(z) = V_p + (z_p - z)\frac{d\ln\rho}{dt}. \qquad (R.21)$$

Eliminating U_{g+} from (R.14) with (R.20) gives

$$\varepsilon \ (U_g - U_p) \ = \ V_p + L \ \frac{d \ln \rho}{dt} \ - \ U_p \qquad (R.22)$$

or

$$U_g \ = \ U_p \ + \ \frac{1}{\varepsilon} \left(V_p + L \ \frac{d \ln \rho}{dt} \ - \ U_p \right). \qquad (R.23)$$

Differentiating (R.23) yields

$$\dot{U}_g \ = \ - \ \dot{U}_p \left(\frac{1-\varepsilon}{\varepsilon} \right) \ - \ \frac{\dot{\varepsilon}}{\varepsilon^2} \left(V_p + L \ \frac{d \ln \rho}{dt} \ - \ U_p \right)$$

$$+ \ \frac{\dot{V}_p}{\varepsilon} \ + \ \frac{L}{\varepsilon} \ \frac{d^2 \ln \rho}{dt^2} \ + \ \frac{\dot{L}}{\varepsilon} \ \frac{d \ln \rho}{dt}. \qquad (R.24)$$

We anticipate the need to factor out the base pressure P_B dependence of both $\dot{V}_p$ and $d^2 \ln\rho/dt^2$.

The projectile accleration is given by

$$\dot{V}_p \ = \ \frac{g_o A_B P_B}{M_p} \ - \ \frac{g_o A_B P_{res}}{M_p}. \qquad (R.25)$$

To get the base pressure P_B dependence associated with $d^2 \ln\rho/dt^2$ we start with the definition of the gas density

$$\rho \ = \ \frac{m}{V_F} \qquad (R.26)$$

where m = mass of gas burnt and V_F = free volume.

Therefore

$$V_F \ = \ V_o \ + \ A_B \ (z_p - z_{po}) \ - \ \frac{C}{\rho_p} \ + \ \frac{m}{\rho_p} \qquad (R.27)$$

and

$$\dot{V}_F \ = \ A_B \dot{z}_p \ + \ \frac{\dot{m}}{\rho_p} \ = \ A_B V_p \ + \ \frac{\dot{m}}{\rho_p} \qquad (R.28)$$

and

$$\ddot{V}_F = A_B \ddot{V}_p + \frac{\ddot{m}}{\rho_p} \tag{R.29}$$

where it is understood that

$$\frac{m}{\rho_p}, \quad \frac{\dot{m}}{\rho_p}, \quad and \quad \frac{\ddot{m}}{\rho_p}$$

may be in the form

$$\Sigma\frac{m_i}{\rho_{pi}}, \quad \Sigma\frac{\dot{m}_i}{\rho_{pi}}, \quad and \quad \Sigma\frac{\ddot{m}_i}{\rho_{pi}}$$

where the i index refers to different propellants.

From (R.26) we get

$$\text{in}\rho = \ln m - \ln V_F \tag{R.30}$$

and

$$\frac{d\ln\rho}{dt} = \frac{\dot{m}}{m} - \frac{\dot{V}_F}{V_F} \tag{R.31}$$

and

$$\frac{d^2\ln\rho}{dt^2} = \frac{\ddot{m}}{m} - \frac{\dot{m}^2}{m^2} - \frac{\ddot{V}_F}{V_F} + \frac{\dot{V}_F^2}{V_F^2}. \tag{R.32}$$

Substituting (R.29) and then (R.25), we get

$$\frac{d^2\ln\rho}{dt^2} = C_1(t) - \frac{g_o A_B^2 P_B}{V_F M_p} + \frac{g_o A_B^2 P_{res}}{M_p V_F} \tag{R.33}$$

where

$$C_1(t) = \ddot{m}\left(\frac{1}{m} - \frac{1}{\rho_p V_F}\right) + \frac{\dot{V}_F^2}{V_F^2} - \frac{\dot{m}^2}{m^2}.$$

We still require $\ddot{m}$, which we get from

$$\dot{m} = \rho_p S \frac{dx}{dt} \tag{R.34}$$

where S is the surface area of the propellant and dx/dt is the burning rate of the propellant where dx/dt is of the form

$$\frac{dx}{dt} = aP_m^n. \qquad (R.35)$$

Therefore, from (R.34) we get

$$\dot{m} = \rho_p \dot{S} \frac{dx}{dt} + \rho_p S \frac{d^2x}{dt^2}$$

$$= \rho_p \frac{dS}{dx}\left(\frac{dx}{dt}\right)^2 + \rho_p S \frac{d^2x}{dt^2} \qquad (R.36)$$

and from (R.35) we get

$$\frac{d^2x}{dt^2} = an\, P_m^{n-1} \frac{dP_m}{dt} \qquad (R.37)$$

where dP_m/dt is determined numerically or from

$$\frac{dP_m}{dt} = \frac{\frac{\dot{m}RT}{m_w} + \frac{\dot{T}Rm}{m_w} - P_m\dot{V}}{V}.$$

Also, defining ϕ as the mass fraction of propellant burned, then

$$\varepsilon\rho = \frac{\phi c - \rho A_B L}{V(z_b)} = \frac{\phi_* C}{V(z_b)} \qquad (R.38)$$

where

$$\phi_* = \phi - \frac{\rho A_B L}{C} \qquad (R.39)$$

and

$$(1-\varepsilon)\ \rho_p = \frac{(1-\phi)C}{V(z_b)}. \qquad (R.40)$$

Differentiating (R.39), we get

$$\dot{\phi}_* = \dot{\phi} - \frac{\rho A_B}{C}\left[\dot{L} + L\frac{d\ln\rho}{dt}\right]. \qquad (R.41)$$

Solving (R.40) for ε, we get

$$\varepsilon = 1 - \frac{(1-\phi)\,C}{\rho_p V(z_b)}. \tag{R.42}$$

Differentiating (R.42) and noting that

$$V(z_b) = V(z_{po}) + (z_b - z_{po})\,A_B \tag{R.43}$$

and therefore

$$\dot{V}(z_b) = \dot{z}_b\,A_B = U_p\,A_B, \tag{R.44}$$

then

$$\dot{\varepsilon} = \frac{\dot{\phi}C}{\rho_p V(z_b)} + \frac{(1-\phi)\;\;C\dot{V}(z_b)}{\rho_p V^2(z_b)} = \frac{\dot{\phi}C}{\rho_p V(z_b)} + \frac{(1-\phi)\;\;C U_p A_B}{\rho_p V^2(z_b)}. \tag{R.45}$$

We now have the quantities which are required for the solution of the momentum equation from which we get the pressure distrubtion in the mixture region.

Consider the momentum equations for each phase

$$\frac{1}{A}\left[\frac{\partial A\varepsilon\rho u}{\partial t} + \frac{\partial A\varepsilon\rho u^2}{\partial z}\right] + \varepsilon\;g_o\;\frac{\partial P}{\partial z} = -\;f_s + \dot{m}\;\;u_p \tag{R.46}$$

$$\frac{1}{A}\left[\frac{\partial A\;\;(1-\varepsilon)\;\;\rho_p u_p}{\partial t} + \frac{\partial A\;\;(1-\varepsilon)\;\;\rho_p u_p^2}{\partial z}\right] + (1-\varepsilon)\;\;g_o\;\frac{\partial P}{\partial z} = f_s - \dot{m}u_p \tag{R.47}$$

where

$$f_s = \frac{(1-\varepsilon)}{D_p}\;\rho\,(u-u_p)^2 f_{sc} \tag{R.48}$$

and

$$f_{sc} = \begin{cases} \dfrac{2.5\lambda}{REN^{.081}}\;f_{so}\;, & \varepsilon < \varepsilon_o \\[2ex] \dfrac{2.5\lambda}{REN^{.081}}\left(\dfrac{(1-\varepsilon)}{(1-\varepsilon_o)}\;\dfrac{\varepsilon_o}{\varepsilon}\right)^{0.45} f_{so}, & \varepsilon \ge \varepsilon_o \end{cases} \tag{R.49}$$

where

$$\lambda = \left(\frac{0.5 + \frac{GL}{GD}}{\left(1.5 \frac{GL}{GD}\right)^{2/3}} \right)^{2.17} \qquad (R.50)$$

and

$$REN = \rho D_p \ |U_g - U_p| \ /\mu.$$

Adding (R.46) and (R.47) we get

$$g_o \frac{\partial P}{\partial z} = - \frac{1}{A} \frac{\partial}{\partial t} [A\varepsilon\rho u + A(1-\varepsilon)\rho_p u_p]$$

$$- \frac{1}{A} \frac{\partial}{\partial z} [A\varepsilon\rho u^2 + A(1-\varepsilon)\rho_p u_p^2]. \qquad (R.51)$$

From (R.8) we get

$$u(z) = \frac{U_g A_B V(z)}{A(z)V(z_b)},$$

from (R.13) we get

$$u_p(z) = \frac{U_p A_B V(z)}{A(z)V(z_b)},$$

from (R.38) we get

$$\varepsilon\rho = \frac{\phi_* C}{V(z_b)},$$

and from (R.40) we get

$$(1-\varepsilon) \ \rho_p = \frac{(1-\phi)C}{V(z_b)}$$

which, when substituted into (R.51), yields

$$g_o \frac{\partial P}{\partial z} = - \frac{1}{A} \frac{\partial}{\partial t} \left[\frac{\phi_* C U_g A_B V(z)}{V^2(z_b)} + \frac{(1-\phi)C U_p A_B V(z)}{V^2(z_b)} \right]$$

$$- \frac{1}{A} \frac{\partial}{\partial z} \left[\frac{\phi_* C U_g^2 A_B^2 V^2(z)}{V^3(z_b)A(z)} + \frac{(1-\phi)C U_p^2 A_B^2 V^2(z)}{V^3(z_b)A(z)} \right], \qquad (R.52)$$

or

$$g_o \frac{\partial P}{\partial t} = -\frac{CA_BV(z)}{A(z)} \frac{\partial}{\partial t}\left[\frac{\phi_\circ U_g}{V^2(z_b)} + \frac{(1-\phi)U_p}{V^2(z_b)}\right]$$

$$-\frac{CA_B^2}{A(z)V^3(z_b)} \frac{\partial}{\partial z}\left[\frac{\phi_\circ U_g^2 V^2(z)}{A(z)} + \frac{(1-\phi)U_p^2V^2(z)}{A(z)}\right]. \quad (R.53)$$

Performing the indicated differentiation in (R.53), we get

$$g_o \frac{\partial P}{\partial z} = -\frac{CA_BV(z)}{A(z)}\left[\frac{\dot{\phi}_\circ U_g}{V^2(z_b)} + \frac{\phi_\circ \dot{U}_g}{V^2(z_b)} - 2\frac{\phi_\circ U_g \dot{V}(z_b)}{V^3(z_b)} - \frac{\dot{\phi} U_p}{V^2(z_b)}\right.$$

$$\left.+\frac{(1-\phi)\dot{U}_p}{V^2(z_b)} - 2\frac{(1-\phi)U_p\dot{V}(z_b)}{V^3(z_b)}\right]$$

$$-\frac{CA_B^2}{A(z)V^3(z_b)}[\phi_\circ U_g^2 + (1-\phi)U_p^2]\frac{\partial}{\partial z}\left(\frac{V^2(z)}{A(z)}\right). \quad (R.54)$$

Noting that, since

$$V(z_b) = V(z_{bo}) + A_B\ (z_b - z_{bo}), \quad (R.55)$$

and therefore

$$\dot{V}(z_b) = A_B\dot{z}_b = A_BU_p, \quad (R.56)$$

and since $\partial V(z)/\partial z = A(z)$, and A(z) and V(z) are function of z only,

$$\frac{\partial}{\partial z}\left(\frac{V^2(z)}{A(z)}\right) = 2V(z) - \frac{V^2(z)}{A^2(z)}\frac{dA(z)}{dz}, \quad (R.57)$$

then

$$g_o \frac{\partial P}{\partial z} = -\frac{CA_BV(z)}{A(z)V^2(z_b)}\left[\dot{\phi}_\circ U_g + \phi_\circ \dot{U}_g - 2\frac{\phi_\circ U_g A_B U_p}{V(z_b)} - \dot{\phi} U_p\right.$$

$$+(1-\phi)\dot{U}_p - 2\frac{(1-\phi)U_p^2A_B}{V(z_b)} + \frac{2A_B}{V(z_b)}(\phi_\circ U_g^2 + (1-\phi)U_p^2)$$

$$\left.-\frac{A_BV(z)}{A^2(z)V(z_b)}(\phi_\circ U_g^2 + (1-\phi)U_p^2)\frac{\partial A(z)}{\partial z}\right] \quad (R.58)$$

or

$$g_o \frac{\partial P}{\partial z} = - \frac{CA_B V(z)}{A(z)V^2(z_b)} \Big[\phi_\circ \dot{U}_g + (1-\phi)\dot{U}_p + \dot{\phi}_\circ U_g - \dot{\phi} U_p$$

$$+ 2 \frac{A_B \phi_\circ U_g}{V(z_b)} (U_g - U_p)$$

$$- \frac{A_B V(z)}{A^2(z)V(z_b)} (\phi_\circ U_g^2 + (1-\phi)U_p^2) \frac{\partial A(z)}{\partial z} \Big]. \qquad (R.59)$$

Eliminating $\dot{U}_g$ from (R.59) by using (R.24), we get

$$g_o \frac{\partial P}{\partial z} = - \frac{CA_B V(z)}{V^2(z_b)A(z)} \left[\frac{\phi_\circ \dot{V}_p}{\varepsilon} + \frac{L\phi_\circ}{\varepsilon} \frac{d^2 \ln\rho}{dt^2} \right.$$

$$+ \dot{U}_p \left(1 - \phi - \frac{\phi_\circ (1-\varepsilon)}{\varepsilon} \right) - \phi_\circ \frac{\dot{\varepsilon}}{\varepsilon^2} \left(V_p + L \frac{d\ln\rho}{dt} - U_p \right)$$

$$+ \phi_\circ \frac{L}{\varepsilon} \frac{d\ln\rho}{dt} + \dot{\phi}_\circ U_g - \dot{\phi} U_p + \frac{2A_B \phi_\circ U_g}{V(z_b)} (U_g - U_p)$$

$$\left. - \frac{A_B V(z)}{V(z_b)A^2(z)} (\phi_\circ U_g^2 + (1-\phi)U_p^2) \frac{\partial A(z)}{\partial z} \right]. \qquad (R.60)$$

We need to eliminate $\dot{U}_p$.

From (R.2) we get, by multiplying by u_p,

$$u_p \rho_p \frac{\partial \varepsilon}{\partial t} - \frac{u_p}{A} \frac{\partial \ (1-\varepsilon) \ \rho_p u_p A}{\partial z} = \dot{m}(t) \ u_p \qquad (R.61)$$

which, when substituted into (R.47), gives

$$\frac{1}{A} \frac{\partial A(1-\varepsilon)\rho_p u_p}{\partial t} + \frac{1}{A} \frac{\partial A(1-\varepsilon)\rho_p u_p^2}{\partial z} + (1-\varepsilon) \ g_o \frac{\partial P}{\partial z}$$

$$= f_s - u_p \rho_p \frac{\partial \varepsilon}{\partial t} + \frac{u_p}{A} \frac{\partial (1-\varepsilon)\rho_p u_p A}{\partial z} \qquad (R.62)$$

or

$$(1-\varepsilon)\rho_p \frac{\partial u_p}{\partial t} + \frac{(1-\varepsilon)\rho_p u_p}{A}\frac{\partial A}{\partial t} + 2(1-\varepsilon)\rho_p \frac{\partial u_p}{\partial z}$$

$$+ \frac{(1-\varepsilon)\rho_p u_p^2}{A}\frac{\partial A}{\partial z} + (1-\varepsilon)g_o \frac{\partial P}{\partial z}$$

$$= f_s + u_p (1-\varepsilon)\rho_p \frac{\partial u_p}{\partial z} + \frac{u_p(1-\varepsilon)\rho_p u_p}{A}\frac{\partial A}{\partial z}. \qquad (R.63)$$

Evaluating R.63 at z_b and noting $\partial A/\partial t = 0$,

$$\frac{\partial U_p}{\partial z}\Big|_{z_b} = 0$$

and

$$\frac{\partial A}{\partial z}\Big|_{z_b} = 0 \quad for \quad z_b \geq z_{po},$$

then

$$(1-\varepsilon) \; g_o \frac{\partial P}{\partial z} + (1-\varepsilon)\rho_p \dot{U}_p = f_s. \qquad (R.63a)$$

Using (R.48) with (R.63a), we get

$$\dot{U}_p = \frac{\rho(U_g - U_p)^2 f_{sc}}{\rho_p D_p} - \frac{g_o}{\rho_p}\frac{\partial P}{\partial z}\Big|_{z_b}. \qquad (R.64)$$

Substituting (R.64) into (R.60) gives

$$
\begin{aligned}
g_o \frac{\partial P}{\partial z} = & - \frac{CA_B V(z)}{V^2(z_b)A(z)} \left[\frac{\phi_* \dot{V}_p}{\varepsilon} + L \frac{\phi_*}{\varepsilon} \frac{d^2 \ln \rho}{dt^2} \right. \\
& + \left(1 - \phi - \frac{\phi_*(1-\varepsilon)}{\varepsilon} \right) \frac{\rho (U_g - U_p)^2}{\rho_p D_p} f_{s_c} \\
& - \left(1 - \phi - \frac{\phi_*(1-\varepsilon)}{\varepsilon} \right) \frac{g_o}{\rho_p} \frac{\partial P}{\partial z}\Big|_{z_b} \\
& - \frac{\dot{\varepsilon}\phi_*}{\varepsilon^2} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) + \frac{L\phi_*}{\varepsilon} \frac{d \ln \rho}{dt} \\
& + \phi_* U_g - \dot{\phi} U_p + \frac{2A_B \phi_* U_g}{V(z_b)} (U_g - U_p) \\
& \left. - \frac{A_B V(z)}{V(z_b)A^2(z)} \left(\frac{\partial A(z)}{\partial z} \right) (\phi_* U_g^2 + (1-\phi)U_p^2) \right]
\end{aligned}
\tag{R.65}
$$

or

$$
\begin{aligned}
g_o \frac{\partial P}{\partial z} = & - \frac{V(z)}{A(z)} \frac{CA_B}{V^2(z_b)} \left[\phi_1 - \phi_2 \frac{g_o}{\rho_p} \frac{\partial P}{\partial z}\Big|_{z_b} \right. \\
& \left. - \frac{\phi_3 A_B V(z)}{V(z_b)A^2(z)} \frac{\partial A(z)}{\partial z} \right]
\end{aligned}
\tag{R.66}
$$

and

$$
\phi_1 = \phi_1' + \frac{L\phi_*}{\varepsilon} \frac{d^2 \ln \rho}{dt^2} + \frac{\phi_* \dot{V}_p}{\varepsilon} \tag{R.67}
$$

and

$$
\begin{aligned}
\phi_1' = & \dot{\phi}_* U_g - \dot{\phi} U_p - \frac{\phi_* \dot{\varepsilon}}{\varepsilon^2} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) + \frac{L\phi_*}{\varepsilon} \frac{d \ln \rho}{dt} \\
& + \frac{2A_B \phi_* U_g (U_g - U_p)}{V(z_b)} + \phi_2 \frac{\rho (U_g - U_p)^2 f_{sc}}{\rho_p D_p}
\end{aligned}
\tag{R.68}
$$

and

$$
\phi_2 = 1 - \phi - \frac{\phi_*(1-\varepsilon)}{\varepsilon} \tag{R.69}
$$

and

$$\phi_3 = \phi_* \ U_g^2 + (1-\phi) \ U_p^2. \tag{R.70}$$

To get

$$\frac{\partial P}{\partial z}\Big|_{z_b}$$

for elimination from (R.66), we evaluate (R.66) at z_b and solve for

$$\frac{\partial P}{\partial z}\Big|_{z_b}.$$

With (R.66) evaluated at z_b and noting

$$\frac{\partial A}{\partial z}\Big|_{z_b} = 0$$

and $A(z_b) = A_B$, we get

$$g_o \frac{\partial P}{\partial z}\Big|_{z_b} = -\frac{C}{V(z_b)}\left[\phi_1 - \phi_2 \frac{g_o}{\rho_p} \frac{\partial P}{\partial z}\Big|_{z_b}\right] \tag{R.71}$$

or

$$\frac{\partial P}{\partial z}\Big|_{z_b} = -\frac{C\phi_1}{g_o V(z_b)\left(1 - \frac{\phi_2 C}{\rho_p V(z_b)}\right)} \quad . \tag{R.72}$$

Substituting (R.72) back into (R.66), we get

$$g_o \frac{\partial P}{\partial z} = -\frac{V(z)}{A(z)} \frac{CA_B}{V^2(z_b)}\left[\phi_1 + \frac{\phi_2 g_o C\phi_1}{\rho_p g_o V(z_b)\left(1 - \frac{\phi_2 C}{\rho_p V(z_b)}\right)} - \frac{\phi_3 A_B V(z)}{V(z_b) A^2(z)} \frac{\partial A(z)}{\partial z}\right] \tag{R.73}$$

or

$$g_o \frac{\partial P}{\partial z} = -\frac{V(z)}{A(z)} \frac{CA_B}{V^2(z_b)}\left[\frac{\phi_1}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} - \frac{\phi_3 A_B V(z)}{V(z_b) A^2(z)} \frac{\partial A(z)}{\partial z}\right]. \tag{R.74}$$

ϕ_1 depends upon the base pressure $P_B = P(z_p)$ (one of the values we wish to solve for) through $\dot{V}_p$ (the accleration of the projectile) and also through $d^2 \ln\rho/dt^2$.

Substituting (R.33) into (R.67), we get

$$\phi_1 = \phi_1' + \frac{L\phi_* C_1(t)}{\varepsilon} - \frac{L\phi_* A_B}{\varepsilon V_F}\dot{V}_p + \frac{\phi_*}{\varepsilon}\dot{V}_p \tag{R.75}$$

or

$$\phi_1 = \phi_1' + D + \frac{E\phi_*}{\varepsilon}\dot{V}_p \tag{R.76}$$

where

$$D = \frac{L\phi_* C_1(t)}{\varepsilon} \tag{R.77}$$

and

$$E = \left(1 - \frac{LA_B}{V_F}\right). \tag{R.78}$$

Substituting (R.25) into (R.76), we get

$$\phi_1 = \phi_1' + D + \frac{E\phi_*}{\varepsilon}\left(\frac{g_o A_B P_B}{M_p} - \frac{g_o A_B P_{res}}{M_p}\right) \tag{R.79}$$

or

$$\phi_1 = \phi_1' + D - \frac{E\phi_* g_o A_B P_{res}}{\varepsilon M_p} + \frac{E\phi_* g_o A_B P_B}{\varepsilon M_p}. \tag{R.80}$$

Substituting (R.80) into (R.74), we get

$$\frac{\partial P}{\partial z} = -\frac{V(z)CA_B}{A(z)g_o V^2(z_b)}\left[\frac{\phi_1' + D - \frac{E\phi_* g_o A_B P_{res}}{\varepsilon M_p} + \frac{E\phi_* g_o A_B P_B}{\varepsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}}\right.$$
$$\left. - \frac{\phi_3 A_B V(z)}{V(z_b)A^2(z)}\frac{\partial A}{\partial z}\right] \tag{R.81}$$

or

$$\frac{\partial P}{\partial z} = \frac{CA_B}{g_o V^2(z_b)} \left[- \frac{\phi_1' + D - \frac{E\phi_4 g_o A_B P_{res}}{\varepsilon M_p} + \frac{E\phi_4 g_o A_B P_B}{\varepsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \frac{V(z)}{A(z)} \right.$$

$$\left. + \frac{\phi_3 A_B V^2(z)}{V(z_b) A^3(z)} \frac{\partial A(z)}{\partial z} \right]. \tag{R.82}$$

Integrating (R.82) and noting that A(z) and V(z) are functions of z and only of z and that

$$\int \frac{V^2(z)}{A^3(z)} \frac{\partial A}{\partial z} \partial z$$

(by parts) can be written

$$- \frac{V^2(z)}{2A^2(z)} + \int \frac{V(z)}{A(z)} dz$$

we get

$$P(z) - P(0) = \frac{CA_B}{g_o V^2(z_b)} \left[- \frac{\phi_1' + D - \frac{E\phi_4 g_o A_B P_{res}}{\varepsilon M_p} + \frac{E\phi_4 g_o A_B P_B}{\varepsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \int \frac{V(z)}{A(z)} dz \right.$$

$$\left. - \frac{\phi_3 A_B V^2(z)}{2V(z_b) A^2(z)} + \frac{\phi_3 A_B}{V(z_b)} \int \frac{V(z)}{A(z)} dz \right]. \tag{R.83}$$

Noting $P(0) = P_{Br}$ and collecting terms, we get

$$P(z) = P_{Br} + \left[\frac{CA_B}{g_o V^2(z_b)} \left(\frac{\phi_3 A_B}{V(z_b)} - \frac{\phi_1' + D - \frac{E\phi_4 g_o A_B P_{rex}}{\varepsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right) \right.$$

$$\left. - \frac{\frac{CE\phi_4 A_B^2 P_B}{V^2(z_b)\varepsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right] \int \frac{V(z)}{A(z)} dz - \frac{\phi_3 C A_B^2 V^2(z)}{2 g_o V^3(z_b) A^2(z)} \tag{R.83a}$$

or

$$P(z) = P_{Br} + (a_1(t) + a_2(t) \ P_B) \ J_1(z) + b(t) \ J_2(z) \tag{R.84}$$

where

$$a_1(t) = \frac{CA_B}{g_o V^2(z_b)} \left(\frac{\phi_3 A_B}{V(z_b)} - \frac{\phi_1' + D - \frac{E\phi_4 g_o A_B P_{rex}}{\varepsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right) \tag{R.85}$$

and

$$a_2(t) = - \frac{\frac{CE\phi_4 A_B^2}{V^2(z_b)\varepsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \tag{R.86}$$

and

$$b(t) = - \frac{C\phi_3 A_B^2}{2 g_o V^3(z_b)} \tag{R.87}$$

and

$$J_1(z) = \int_o^z \frac{V(z)}{A(z)} dz \tag{R.88}$$

and

$$J_2(z) = \frac{V^2(z)}{A^2(z)}. \tag{R.89}$$

(R.84) is the equation for the pressure distribution in the mixture region. To complete the pressure disuribution description, we require the distribution in the ullage region which should be identical to the description in the velocity lag derivation that is in the ullage region. The momentum equation for the gas is

$$\frac{\partial P}{\partial z} = - \frac{\rho}{g_o} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right). \tag{R.90}$$

The equation of continuity in the ullage region is

$$\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{d\ln\rho}{dt} \tag{R.91}$$

Integrating (R.91) and noting $d\ln\rho/dt$ is a function of time only we get

$$u(z) = -\frac{d\ln\rho}{dt}\, z + f(t). \tag{R.92}$$

With the boundary condition $u(z_p) = V_p$ we get

$$V_p = -z_p \frac{d\ln\rho}{dt} + f(t) \tag{R.93}$$

and therefore,

$$f(t) = V_p + z_p \frac{d\ln\rho}{dt} \tag{R.94}$$

and

$$u(z) = V_p + (z_p - z)\frac{d\ln\rho}{dt}. \tag{R.95}$$

Differentiating (R.95) and noting $\dot{z}_p = V_p$ gives

$$\frac{\partial u}{\partial t} = \dot{V}_p + (z_p - z)\frac{d^2\ln\rho}{dt^2} + V_p\frac{d\ln\rho}{dt}. \tag{R.96}$$

Substituting (R.91), (R.95), and (R.96) into (R.90), we get

$$\frac{\partial P}{\partial z} = -\frac{\rho}{g_o}\left[\dot{V}_p + (z_p - z)\frac{d^2\ln\rho}{dt^2} + V_p\frac{d\ln\rho}{dt} + \left(V_p + (z_p - z)\frac{d\ln\rho}{dt}\right)\left(-\frac{d\ln\rho}{dt}\right)\right. \tag{R.97}$$

or

$$\frac{\partial P}{\partial z} = -\frac{\rho}{g_o}\left[\dot{V}_p + (z_p - z)\frac{d^2\ln\rho}{dt^2} - (z_p - z)\left(\frac{d\ln\rho}{dt}\right)^2\right]. \tag{R.98}$$

Integrating (R.98) from z_b to z and noting z_p, $\dot{V}_p$, and $\ln\rho$ are functions of time only, we get

$$P(z) = P(z_b) - \frac{\rho}{g_o}\left[\dot{V}_p z - \frac{(z_p - z)^2}{2}\frac{d^2\ln\rho}{dt^2} + \frac{(z_p - z)^2}{2}\left(\frac{d\ln\rho}{dt}\right)^2\right]_{z_b}^{z} \tag{R.99}$$

or

$$P(z) = P(z_b) - \frac{\rho}{g_o}\left[\dot{V}_p z - \frac{(z_p - z)^2}{2} \Delta\right]\Big|_{z_b}^{z} \qquad (R.100)$$

where

$$\Delta = \frac{d^2 \ln\rho}{dt^2} - \left(\frac{d ln\rho}{dt}\right)^2. \qquad (R.101)$$

Therefore, we get the pressure distribution in the ullage region as

$$P(z) = P(z_b) - \frac{\rho}{g_o}\left[\dot{V}_p \ (z - z_b) + ((z_p - z_b)^2 - (z_p - z)^2) \frac{\Delta}{2}\right] \qquad (R.102)$$

or

$$P(z) = P(z_b) - \frac{\rho}{g_o}\left[(\dot{V}_p + z_p\Delta) \ (z - z_b) - (z^2 - z_b^2) \frac{\Delta}{2}\right]. \qquad (R.103)$$

Defining the mean pressure P_m to be

$$P_m = \frac{\int_o^{z_p} P(z) A(z) dz}{\int_o^{z_p} A(z) dz} \qquad (R.104)$$

and since $\int_o^{z_p} A(z)dz = V(z_p)$ and using (R.84) and (R.103)

$$P_m = \frac{\int_o^{z_b} (P_{Br} + a_1(t) \ J_1(z) + a_2(t) \ P_B \ J_1(z) + b(t) \ J_2(z)) \ A(z)dz}{V(z_p)}$$

$$+ \frac{\int_{z_b}^{z_p} \left(P(z_b) - \frac{\rho}{g_o}\left[(\dot{V}_p + z_p \ \Delta) \ (z - z_b) - (z^2 - z_b^2) \ \frac{\Delta}{2} \right]\right) A(z)dz}{V(z_p)}$$

$$(R.105)$$

or

$$P_m = \frac{P_{Br} \ V(z_b) + a_1(t) \ J_3(z_b) + a_2(t) \ P_B J_3(z_b) + b(t) \ J_4(z_b)}{V(z_b)}$$

$$+ \frac{A_B L P(z_b)}{V(z_p)} - \frac{A_B \rho}{g_o}\left[\frac{\frac{(\dot{V}_p + z_p\Delta) \ (z_p - z_b)^2}{2} - \frac{\Delta}{6} \ (z_p^3 - z_b^3)}{V(z_p)}\right.$$

$$\left. + \frac{\frac{z_b^2\Delta \ (z_p - z_b)}{2}}{V(z_p)}\right] \qquad (R.106)$$

or

$$P_m = P_{Br} \frac{V(z_b)}{V(z_p)} + \frac{a_1(t) \quad J_3(z_b)}{V(z_p)} + \frac{a_2(t) \quad P_B J_3(z_b)}{V(z_p)} + \frac{b(t) \quad J_4(z_b)}{V(z_p)}$$

$$+ \frac{A_B L P(z_b)}{V(z_p)} - \frac{\rho A_B}{g_o} \left[\dot{V}_p + \frac{2L\Delta}{3} \right] \frac{L^2}{2V(z_p)} \tag{R.107}$$

where

$$J_3(z_b) = \int_o^{z_b} A(z) \quad J_1(z) \quad dz \tag{R.108}$$

and

$$J_4(z_b) = \int_o^{z_b} \frac{V^2(z)}{A(z)} \, dz. \tag{R.109}$$

Substituting in for $\dot{V}_p$ from (R.25) into (R.107), we get

$$P_m = P_{Br} \frac{V(z_b)}{V(z_p)} + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{a_2(t) J_3(z_b) P_B}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)}$$

$$+ \frac{A_B L P(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2 P_B}{2V(z_p) M_p} + \frac{\rho A_B^2 L^2 P_{res}}{2V(z_p) M_p} - \frac{\rho A_B L^3 \Delta}{3 g_o V(z_p)}. \tag{R.110}$$

To eliminate $P(z_b)$, we evaluate (R.84) at z_b and get

$$P(z_b) = P_{Br} + (a_1(t) + a_2(t) \quad P_B) \quad J_1(z_b) + b(t) \quad J_2(z_b) \tag{R.111}$$

and substitute into (R.110) to get

$$P_m = P_{Br} + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{a_2(t) J_3(z_b) P_B}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)}$$

$$+ \frac{A_B L a_1(t) J_1(z_b)}{V(z_p)} + \frac{A_B L a_2(t) J_1(z_b) P_B}{V(z_p)} + \frac{A_B L b(t) J_2(z_b)}{V(z_p)}$$

$$- \frac{\rho A_B^2 L^2 P_B}{2V(z_p) M_p} + \frac{\rho A_B^2 L^2 P_{res}}{2V(z_p) M_p} - \frac{\rho A_B L^3 \Delta}{3 g_o V(z_p)} \tag{R.112}$$

or

$$P_m = P_{Br} + P_B \left(\frac{A_B L a_2(t) J_1(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2}{2V(z_p) M_p} + \frac{a_2(t) J_3(z_b)}{V(z_p)} \right)$$

$$+ \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)} + \frac{A_B L a_1(t) J_1(z_b)}{V(z_p)}$$

$$+ \frac{A_B L b(t) J_2(z_b)}{V(z_p)} + \frac{\rho A_B^2 L^2 P_{res}}{2V(z_p) M_p} - \frac{\rho A_B L^3 \Delta}{3 g_o V(z_p)}. \tag{R.113}$$

We now need P_{Br} in terms of P_B to eliminate P_{Br} from (R.113) and give us an equation for P_B.

Evaluating (R.103) at z_p, we get

$$P(z_p) = P_B = P(z_b) - \frac{\rho}{g_o}\left[(\dot{V}_p + z_p\Delta)\ L - (z_p^2 - z_b^2)\ \frac{\Delta}{2}\right] \qquad (R.114)$$

or, since $z_p^2 - z_b^2 = L\,(z_p + z_b)$,

$$P_B = P(z_b) - \frac{\rho L}{g_o}\left(\dot{V}_p + \frac{L\Delta}{2}\right), \qquad (R.115)$$

and substituting (R.25) for $\dot{V}_p$, we get

$$P_B = P(z_b) - \frac{\rho L A_B P_B}{M_p} + \frac{A_B \rho L P_{res}}{M_p} - \frac{\rho L^2 \Delta}{2g_o}. \qquad (R.116)$$

Substituting (R.111) for $P(z_b)$ into (R.116) and collecting terms, we get

$$0 = P_{Br} + P_B\left(-1 + a_2(t)J_1(z_b) - \frac{\rho L A_B}{M_p}\right) + a_1(t)J_1(z_b)$$
$$+\ b(t)J_2(z_b) + \frac{A_B \rho L P_{res}}{M_p} - \frac{\rho L^2 \Delta}{2g_o}. \qquad (R.117)$$

Subtracting (R.117) from (R.113) eliminates P_{Br} and gives P_B in terms of P_m or

$$P_m = P_B\left[1 - a_2(t)J_1(z_b) + \frac{\rho L A_B}{M_p} + \frac{A_B L a_2(t)J_1(z_b)}{V(z_p)}\right.$$
$$\left. - \frac{\rho A_B^2 L^2}{2V(z_p)M_p} + \frac{a_2(t)J_3(z_b)}{V(z_p)}\right] + \frac{a_1(t)J_3(z_b)}{V(z_p)} + \frac{b(t)J_4(z_b)}{V(z_p)}$$
$$+ \frac{A_B L a_1(t)J_1(z_b)}{V(z_p)} + \frac{A_B L b(t)J_2(z_b)}{V(z_p)} + \frac{\rho A_B^2 L^2 P_{res}}{2V(z_p)M_p} - \frac{\rho A_B L^3 \Delta}{3g_o V(z_p)}$$
$$- a_1(t)J_1(z_b) - b(t)J_2(z_b) - \frac{A_B \rho L P_{res}}{M_p} + \frac{\rho L^2 \Delta}{2g_o}. \qquad (R.118)$$

Since Δ has a dependence on P_B through $d^2 ln\rho/dt^2$ using (R.33) with (R.117), we get

$$P_{Br} = P_B \left(1 - a_2(t)J_1(z_b) + \frac{\rho L A_B}{M_p} - \frac{\rho L^2 A_B^2}{2V_F M_p}\right)$$
$$+ \frac{\rho L^2}{2g_o}\left(C_1(t) - \left(\frac{dln\rho}{dt}\right)^2 + \frac{g_o A_B^2 P_{res}}{M_p V_F}\right) - a_1(t)J_1(z_b)$$
$$- b(t)J_2(z_b) - \frac{A_B \rho L P_{res}}{M_p}, \tag{R.119}$$

and using (R.33) with (R.118), we get

$$P_m = P_B \left[1 + \frac{A_B L a_2(t)J_1(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2}{2V(z_p)M_p} + \frac{a_2(t)J_3(z_b)}{V(z_p)}\right.$$
$$\left. - a_2(t)J_1(z_b) + \frac{\rho L A_B}{M_p} - \frac{\rho L^2 A_B^2}{2V_F M_p} + \frac{\rho A_B^3 L^3}{3V(z_p)V_F M_p}\right]$$
$$+ \frac{\rho L^3}{2g_o}\left(C_1(t) - \left(\frac{dln\rho}{dt}\right)^2 + \frac{g_o A_B^2 P_{res}}{M_p V_F}\right)\left(1 - \frac{2A_B L}{3V(z_p)}\right)$$
$$+ \frac{a_1(t)J_3(z_b)}{V(z_p)} + \frac{b(t)J_4(z_b)}{V(z_p)} + \frac{A_B L a_1(t)J_1(z_b)}{V(z_p)}$$
$$+ \frac{A_B L b(t)J_2(z_b)}{V(z_p)} + \frac{\rho A_B^2 L^2 P_{res}}{2V(z_p)M_p}$$
$$- a_1(t)J_1(z_b) - b(t)J_2(z_b) - \frac{A_B \rho L P_{res}}{M_p}. \tag{R.120}$$

Equation (R.120) gives P_B in terms of P_m (which is determined using an equation of state) and then (R.119) gives P_{Br} from P_B.

The evaluation of $J_1(z_b)$, $J_2(z_b)$, $J_3(z_b)$, and $J_4(z_b)$ can be simplified by noting that the variation in area is confined to the chamber and, therefore,

$$V(z) = V(z_{bo}) + A_B(z - z_{bo}), \quad for \quad z \geq z_{bo}, \tag{R.121}$$

and

$$J_1(z_b) = \int_o^{z_b} \frac{V(z)}{A(z)}\, dz = \int_o^{z_{bo}} \frac{V(z)}{A(z)}\, dz + \int_{z_{bo}}^{z_b} \frac{V(z)}{A(z)}\, dz$$
$$= J_1(z_{bo}) + \int_{z_{bo}}^{z_b} \frac{V(z_{bo}) + A_B(z - z_{bo})}{A_B}\, dz$$
$$= J_1(z_{bo}) + \frac{1}{A_B}\left(V(z_{bo})\ (z_b - z_{bo}) + \frac{A_B(z_b - z_{bo})^2}{2}\right) \tag{R.122}$$

$$J_2(z_b) = \frac{V^2(z)}{A^2(z)}\Big|_{z_b} = \frac{[V(z_{bo}) + A_B \ (z_b - z_{bo})]^2}{A_B^2}, \qquad (R.123)$$

$$\begin{aligned} J_3(z_b) &= \int_o^{z_b} \left(\int_o^z \frac{V(x)}{A(x)} dx \right) A(z) dz \\ &= \int_o^{z_{bo}} \left(\int_o^z \frac{V(x)}{A(x)} dx \right) A(z) dz + \int_{z_{bo}}^{z_b} \left(\int_o^z \frac{V(x)}{A(x)} dx \right) A_B dz \\ &= J_3(z_{bo}) + \int_{z_{bo}}^{z_b} \left(J_1(z_{bo}) + \frac{1}{A_B} \left[V(z_{bo}) \ (z - z_{bo}) + \frac{A_B}{2} \ (z - z_{bo})^2 \right] \right) A_B dz \\ &= J_3(z_{bo}) + A_B J_1(z_{bo}) \ (z_o - z_{bo}) + \frac{V(z_{bo}) \ (z_b - z_{bo})^2}{2} \\ &+ \frac{A_B}{6} \ (z_b - z_{bo})^3, \end{aligned} \qquad (R.124)$$

$$\begin{aligned} J_4(z_b) &= \int_o^{z_b} \frac{V^2(z)}{A(z)} dz = \int_o^{z_{bo}} \frac{V^2(z)}{A(z)} dz + \int_{z_{bo}}^{z_b} \frac{[V(z_{bo}) + A_B \ (z - z_{bo})]^2}{A_B} dz \\ &= J_4(z_{bo}) + \frac{[V(z_{bo}) + A_B \ (z_b - z_{bo})]^3 - V^3 \ (z_{bo})}{3A_B^2}. \end{aligned} \qquad (R.125)$$

Equations (R.122) - (R.125) require the evaluation of the integrals $J_1(z_{bo})$ - $J_4(z_{bo})$ only once.

The kinetic energy (KE) of the gas and solid will be required. The kinetic energy is given by

$$g_o \ KE = \frac{1}{2} \int_o^{z_p} u^2 \ dm$$

where u is the velocity and dm is the differential of mass, and since ρdV = dm and dV = A(z)dz, then

$$\begin{aligned} g_o \ KE &= \frac{1}{2} \ (1 - \varepsilon) \int_o^{z_b} u_p^2 \rho_p A(z) dz \\ &+ \frac{1}{2} \ \varepsilon \int_o^{z_b} u^2 \rho A(z) dz + \frac{1}{2} \int_{z_b}^{z_p} u^2 \rho A_B dz. \end{aligned} \qquad (R.126)$$

We get u in the mixture region from (R.8) and u_p from (R.13) and u in the ullage region from (R.21), and therefore

$$g_o \; KE = \frac{1}{2} (1-\varepsilon) \int_o^{z_b} \frac{U_p^2 A_B^2 \rho_p V^2(z)}{V^2(z_b) A(z)} \, dz$$

$$+ \frac{1}{2} \varepsilon \int_o^{z_b} \frac{U_g^2 A_B^2 \rho_p V^2(z)}{V^2(z_b) A(z)} \, dz$$

$$+ \frac{\rho A_B}{2} \int_{z_b}^{z_p} \left(V_p + \frac{dln\rho}{dt} (z_p - z) \right)^2 dz \qquad (R.127)$$

or

$$KE = \frac{(1-\varepsilon) \; U_p^2 A_B^2 \rho_p J_4(z_b)}{2 g_o V^2(z_b)} + \frac{\varepsilon U_g^2 A_B^2 \rho J_4(z_b)}{2 g_o V^2(z_b)}$$

$$+ \frac{\rho A_B L}{6 g_o} \left[3V_p^2 + 3V_p \; L \; \frac{dln\rho}{dt} + L^2 \left(\frac{dln\rho}{dt} \right)^2 \right]. \qquad (R.128)$$

It would appear that the pressure used in evaluating the burning rate should be the mean pressure over the region occupied by the propellant.

The mean pressure over the mixture region is

$$P_{mix} = \frac{\int_o^{z_b} P(z) A(z) dz}{\int_o^{z_b} A(z) dz} \qquad (R.129)$$

and using (R.84), we get

$$P_{mix} = \frac{\int_o^{z_b} [P_{Br} + (a_1(t) + a_2(t) P_B) \; J_1(z) + b(t) J_2(z)] \; A(z) dz}{V(z_b)} \qquad (R.130)$$

or

$$P_{mix} = P_{Br} + \frac{(a_1(t) + a_2(t) P_B)}{V(z_b)} J_3(z_b) + \frac{b(t)}{V(z_b)} J_4(z_b). \qquad (R.131)$$

INTENTIONALLY LEFT BLANK.

APPENDIX 4:
INPUT DESCRIPTION FOR IBRGA

INTENTIONALLY LEFT BLANK.

USER'S MANUAL FOR IBRGA

IBRGA relies on an input data base consisting of all numerical parameters essential for running the code. All values are in metric units. Below is a compilation of a typical data base showing the name and location of each parameter. The names for the numerical values are prefixed with an alphabetical designator corresponding to the position at which the data are to appear, that is, from left to right. The data may be separated by blanks or commas. The units are shown to the right of each input.

A B C D E F G H I J K

record 1

- A. - chamber volume (cm^3)
- B. - groove diameter (cm)
- C. - land diameter (cm)
- D. - groove/land ratio (none)
  (land, groove, and groove/land ratio used to calculate the projectile bore area)
- E. - twist (turns/caliber)
- F. - projectile travel (cm)
- G. - gradient switch (none)
  (integer value designating the gradient equation to use
  1 = Lagrange, 2 = Chambrage, 3 = Two-phase, 4 = RGA)
- H. - friction factor (none)
  (normally 1 for granular, 0.01 for stick, and 0.1 for partially cut propellant, only used with the two-phase and RGA gradient)

record 1a (Read if and only if gradient switch = 2 or 4)

- A. - number of pairs of points to describe chamber geometry, integer I <= 5 (none)
- B. - initial distance from breech (cm)
  (must be 0.0)
- C. - diameter at initial distance (cm)

.

.

.

- . - Ith distance from breech (cm)
  (initial position of the base of the projectile)
- . - Ith diameter at Ith distance (cm)
  (used to calculate bore area overriding record 1 groove and land diameter specifications)
  (note chamber geometry is used to calculate the chamber volume overriding record 1 chamber volume description)

record 2

- A. - projectile mass (kg)
- B. - switch to calculate energy lost to air resistance,
  integer (none) (0 = no loss, 1= loss)
- C. - fraction of bore resistance work used to heat the tube (none) (>= 0.0, <=1.0)
- D. - gas pressure in front of the projectile (Pa)

record 3

A. - number of pairs of barrel resistance points (none) (integer J <= 10)
B. - bore resistance (MPa)
C. - travel (cm)

.

.

.

. - Jth bore resistance (MPa)
. - Jth travel (cm)

record 4

A. - mass of recoiling parts (kg)
B. - number of recoil point pairs (none) (must be an integer = 2)
C. - recoil force (N) (force to overcome before start of recoil - rod preload)
D. - time of rod preload (s) (must be 0.0)
E. - recoil force (N) (constant resistive force after rise time)
F. - rise time (s) (time to go from start of recoil to constant resistive recoil force)

record 5

A. - free convective heat transfer coefficient (W/cm^2-K)
B. - chamber wall thickness (cm) (wall depth which is heated uniformly)
C. - heat capacity of chamber wall (J/g-K)
D. - initial temperature of chamber wall (K)
E. - heat loss coefficient (none) (usually 1, but may be set to 0.0 to eliminate heat loss)
F. - density of chamber wall (g/cm^3)

record 6

A. - impetus of igniter (J/g)
B. - covolume of igniter (cm^3/g)
C. - adiabatic flame temperature of igniter (K)
D. - mass of igniter (kg)
E. - ratio of specific heats of igniter (none)

record 7

A. - number of propellants (none) (integer <= 10)

record 8

A. - impetus of propellant (J/g)
B. - adiabatic flame temperature of propellant (K)
C. - covolume of propellant (cm^3/g)
D. - mass of propellant (kg)
E. - density of propellant (g/cm^3)
F. - ratio of specific heats of propellant (none)
G. - propellant form function indicator (none)
(integer, may be
0, zero perforated cylindrical grain
1, one perforated cylindrical grain
7, seven perforated cylindrical grain
15, nineteen perforated hexagonal grain
19, nineteen perforated cylindrical grain)
H. - length of propellant grain (cm)
I. - diameter of perforations in the propellant grains (cm) (value ignored if not required but must be present)

J. - outside diameter of propellant grain (cm)
(for the hexagonal grain, it is the distance between rounded corners)

record 8 repeated for each propellant

record 9

A. - number of burning rate triplet points (none)
(integer J <= 10)
B. - exponent (none)
C. - coefficient (cm/s-MPa^{a})
D. - pressure (MPa) (upper pressure limit for which the previous exponent and coefficient are usable)

.
.
.

. - Jth exponent (none)
. - Jth coefficient (cm/s-MPa^{a})
. - Jth pressure (MPa) (if pressure exceeds this limit, then this burning rate equation is used)

record 9 repeated for each propellant

record 10

A. - integration time increment (ms)
B. - print increment (ms)
C. - upper limit on integration time to stop calculation (ms)

INTENTIONALLY LEFT BLANK.

APPENDIX 5:
FORTRAN LISTING OF IBRGA

INTENTIONALLY LEFT BLANK.

```
      program ibrga
      common nsl1,kpr,fracsl(10),dsdxsl(10),surfsl(10),
     & nslp(10),tsl(10),pbrch,pbase,pmean,bbr(10),abr(10),
     & deltat,y(20),igrad
      character outfil*10
      character bdfile*10
      dimension br(10),trav(10),rp(10),tr(10),forcp(10),tempp(10),covp(
     &10)
      dimension chwp(10),rhop(10),gamap(10),nperfs(10),glenp(10),pdp(10
     &),gdiap(10),alpha(10,10),beta(10,10),pres(10,10)
      dimension a(4),b(4),ak(4),d(20),p(20),z(20),frac(10),surf(10
     &),volp(10),dsdx(10),nbr(10),ibo(10),tbo(10),d2xdt2(10),tng(10)
      real lambda,j1zp,j2zp,j3zp,j4zp,j1zb,j2zb,j3zb,j4zb
      dimension chdist(5),chdiam(5),bint(4)
      dimension nsl(10),surfo(10),dsdxn(10)
c     call gettim(ihr,imin,isec,ihuns)
      pi=3.14159
      write(* 15)
15    format(' input name of data file to be used as input ')
      read(*,10)bdfile
10    format(a10)
      open(unit=2,err=999,file=bdfile,status='old',iostat=ios)
      rewind 2
      write(*,25)
25    format(' input name of output file ')
      read(*,10)outfil
      open(unit=6,err=998,file=outfil)
      write(6,16)bdfile
16    format(' the input file is ',a10)
      do 9 i=1,20
      p(i)=0.
      y(i)=0.
      z(i)=0.
      d(i)=0.
9     continue
      read(2,*,end=20,err=30)cham,grvc,aland,glr,twst,travp,igrad
     &,fs0
      if(igrad.gt.1)go to 51
      write(6,55)
55    format(1x,'using Lagrange pressure gradient')
      igrad=1
      go to 52
c     define chambrage assumes nchpts=number of points to define
c     chamber > or = 2 < or = 5 (?),chdiam(I) defines chamber diameter
c     at chdist (I) chamber distance.  chdiam(nchpts) is assumed to be
c     the bore diameter and chdist(i) is assumed to be 0, i.e. at the
c     breech. Assumes truncated cones.
51    if(igrad.eq.3)go to 401
      if(igrad.eq.4)go to 434
      write(6,47,err=30)

47    format(1x,'Using chambrage pressure gradient')
      go to 436
```

```
434   write(6,437)
437   format(1x,'using rga gradient')
      go to 436

436   read(2,*,end=20,err=30)nchpts,(chdist(I),chdiam(I),I=1,nchpts)
      write(6,53,err=30)(chdist(I),chdiam(I),I=1,nchpts)
53    format(///,'   chamber distance cm    chamber diameter cm',/(5x,e14
     &.6,5x,e14.6))
      do 54 I=1,nchpts
      chdist(I)=0.01*chdist(I)
54    chdiam(I)=0.01*chdiam(I)
c     calculate chamber integrals and volume
      if(nchpts.gt.5) write(6,44,err=30)
44    format(1x,'use first 5 points')
      if(nchpts.gt.5)nchpts=5
      bore=chdiam(nchpts)
      if(chdist(1).ne.0.0)write(6,45,err=30)
45    format(1x,' # points ? ')
      chdist(1)=0.0
      pi3=pi/3.0
      b1=0.0
      b2=0.0
      b3=0.0
      b4=0.0
      points=25.0
56    points=points+points
      step=chdist(nchpts)/points
      zz=0.0
      bint(1)=0.0
      bint(3)=0.0
      bint(4)=0.0
      bvol=0.0
      r2=0.5*chdiam(1)
      k=1
      j=int(points+0.5)
      do 57 I=1,j
      zz=zz+step
      if(k.eq.nchpts-1)go to 46
      do 58 I1=k,nchpts-1
      if(zz.gt.chdist(I1).and. zz.lt.chdist(I1+1))go to 59
58    continue
      I1=nchpts-1
59    k=I1
46    diam=(zz-chdist(k))/(chdist(k+1)-chdist(k))
      diam=chdiam(k)+diam*(chdiam(k+1)-chdiam(k))
      r1=0.5*diam
      area=pi*(r1+r2)*(r1+r2)/4.
      bvol=bvol+step*pi3*(r1*r1+r1*r2+r2*r2)
      bint(1)=bint(1)+step*bvol/area
      bint(3)=bint(3)+step*area*bint(1)
      bint(4)=bint(4)+step*bvol*bvol/area
57    r2=r1
      temp=abs(1.0-b1/bint(1))
      if(abs(1.0-b3/bint(3)).gt.temp)temp=abs(1.0-b3/bint(3))
```

```
      if(abs(1.0-b4/bint(4)).gt.temp)temp=abs(1.0-b4/bint(4))
      if(temp.le.0.001)go to 41
      b1=bint(1)
      b3=bint(3)
      b4=bint(4)
      go to 56
41    cham=bvol*1.e6
c     write(6,47,err=30)bint(1),bint(3),bint(4)
c     format(1x,'bint 1 = ',e14.6,'  bint 3 = ',e14.6,'  bint 4 = ',e14.
c    &6)
      chmlen=chdist(nchpts)
      go to 52
 401  write(6,402)
 402  format(1x,'using 2 phase gradient equation')
      goto 52
52    write(6,40,err=30)cham,grve,aland,glr,twst,travp,igrad,fs0
40    format(1x,'chamber volume cm**3',e14.6,/'  groove diam cm',e14.6,/
     &' land diam cm',e14.6,/'  groove/land ratio',e14.6,/'  twist turns
     &/caliber ',e14.6,/'  projectile travel cm',e14.6
     &/'  gradient # ',i3,/' friction factor ',e14.6///)
      cham=cham*1.e-6
      grve=grve*1.e-2
      aland=aland*1.e-2
      travp=travp*1.e-2
      read(2,*,end=20,err=30)prwt,iair,htfr,pgas
      write(6,50,err=30)prwt,iair,htfr,pgas
50    format(1x,'projectile mass kg',e14.6,/'  switch to calculate energ
     &y lost to air resistance J',i2,/'  fraction of work against bore u
     &sed to heat the tube',e14.6/1x,' gas pressure Pa' ,e14.6)
      read(2,*,end=20,err=30)npts,(br(i),trav(i),i=1,npts)
      write(6,60,err=30)npts,(br(i),trav(i),i=1,npts)
60    format(1x,'number barrel resistance points',i2,/'  bore resistance
     & MPa - travel cm'/(1x,e14.6,e14.6))
      write(6,65)
      do 62 i=1,npts
      br(i)=br(i)*1.e6
      trav(i)=trav(i)*1.e-2
62    continue
65    format(1x)
      read(2,*,end=20,err=30)rcwt,nrp,(rp(i),tr(i),i=1,nrp)
      write(6,70,err=30)rcwt,nrp,(rp(i),tr(i),i=1,nrp)
70    format(1x,'  mass of recoiling parts kg',e14.6,/'  number of recoi
     &l point pairs',i2,/'  recoil force N',' recoil time sec'/,(1x,e14
     &.6,3x,e14.6))
      write(6,65)
      read(2,*,end=20,err=30)ho,tshl,cshl,twal,hl,rhocs
      write(6,75,err=30)ho,tshl,cshl,twal,hl,rhocs
75    format(1x,'  free convective heat transfer coefficient w/cm**2 K',
     &e14.6,/'  chamber wall thickness cm',e14.6,/'  heat capacity of st
     &eel of chamber wall J/g K',e14.6,/'  initial temperature of chambe
     &r wall K',e14.6,/'  heat loss coefficient',e14.6,/'  density of ch
     &amber wall steel g/cm**3',e14.6//)
      ho=ho/1.e-4
      tshl=tshl*1.e-2
```

```
      cshl=cshl*1.e+3
      rhocs=rhocs*1.e-3/1.e-6
      read(2,*,end=20,err=30)forcig,covi,tempi,chwi,gamai
      write(6,85,err=30)forcig,covi,tempi,chwi,gamai
85    format(1x,' impetus of igniter propellant J/g',e14.6,/' covolume
     & of igniter cm**3/g',e14.6,/' adiabatic flame temperature of igni
     &ter propellant K',e14.6,/' initial mass of igniter kg',e14.6,/' r
     &atio of specific heats for igniter',e14.6//)
      forcig=forcig*1.e+3
      covi=covi*1.e-6/1.e-3
      read(2,*)nprop
      write(6,98)nprop
98    format(' there are ',i2,' propellants')
      read(2,*,end=20,err=30)(forcp(i),tempp(i),covp(i),chwp(i),
     &rhop(i),gamap(i),nperfs(i),glenp(i),pdp(i),gdiap(i),i=1,nprop)
      write(6,95,err=30)(i,forcp(i),tempp(i),covp(i),chwp(i)
     &,rhop(i),gamap(i),nperfs(i),glenp(i),pdp(i),gdiap(i),I=1,nprop)
95    format((' for propellant number',i2,/' impetus of propellant J/g
     &',e14.6,/' adiabatic temperature of propellant K',e14.6,/' covol
     &ume of propellant cm**3/g',e14.6/' initial mass of propellant kg'
     &,e14.6/' density of propellant g/cm**3',e14.6/' ratio of specifi
     &c heats for propellant',e14.6/' number of perforations of propell
     &ant',i2/' length of propellant grain cm',e14.6/' diameter of per
     &foration in propellant grains cm',e14.6/' outside diameter of pro
     &pellant grain cm',e14.6/)//)
      tmpi=0.0
      do 96 i=1,nprop
      forcp(i)=forcp(i)*1.e+3
      covp(i)=covp(i)*1.e-6/1.e-3
      rhop(i)=rhop(i)*1.e-3/1.e-6
      glenp(i)=glenp(i)*0.01
      pdp(i)=pdp(i)*0.01
      gdiap(i)=gdiap(i)*0.01
      tmpi=tmpi+chwp(i)
      kpr=i
      call prf710(pdp(i),gdiap(i),glenp(i),nperfs(i),0.,
     & frac(i),volp(i),surf(i),dsdx(i))
      tng(i)=chwp(i)/rhop(i)/volp(i)
      surfo(i)=surf(i)
      write(6,408)i,tng(i)
408   format(' for propellant ',i2,' the total number of grains'
     & ,' is ',e14.6)
96    continue
      tmpi=tmpi+chwi
      do 97 j=1,nprop
      read(2,*,end=20,err=30)nbr(j),(alpha(j,i),beta(j,i),pres(j,i),
     &i=1,nbr(j))
      write(6,110,err=30)nbr(j),(alpha(j,i),beta(j,i),pres(j,i),
     &i=1,nbr(j))
110   format(1x,'number of burning rate points',i2/3x,' exponent',8x,'
     & coefficient',10x,' pressure'/5x,'-',15x,'cm/sec-MPa**ai',10x,'MP
     &a',/(1x,e14.6,5x,e14.6,15x,e14.6))
      do 112 i=1,nbr(j)
      beta(j,i)=beta(j,i)*1.e-2
```

```
      pres(j,i)=pres(j,i)*1.e6
112   continue
97    continue
      write(6,65)
      read(2,*,end=20,err=30)deltat,deltap,tstop
      write(6,120,err=30)deltat,deltap,tstop
120   format(1x,'time increment msec',e14.6,'  print increment msec',e14
     &.6/1x,'time to stop calculation msec ',e14.6)
      write(*,130)
      deltat=deltat*0.001
      deltap=deltap*0.001
      tstop=tstop*.001
130   format(1x,'end input data -- I.B. calculation start')
      if(igrad.eq.2.or.igrad.eq.4)go to 131
      bore=(glr*grve*grve+aland*aland)/(glr+1.)
      bore=sqrt(bore)
131   areab=pi*bore*bore/4.
      lambda=1./((13.2+4.*log10(100.*bore))**2)
      iplot=0
      pltdt=deltat
      pltt=0.
      pmaxm=0.0
      pmaxbr=0.0
      pmaxba=0.0
      tpmaxm=0.0
      tpmxbr=0.0
      tpmxba=0.0
      tpmax=0.0
      a(1)=0.5
      a(2)=1.-sqrt(2.)/2.
      a(3)=1.+sqrt(2.)/2.
      a(4)=1./6.
      b(1)=2.
      b(2)=1.
      b(3)=1.
      b(4)=2.
      ak(1)=0.5
      ak(2)=a(2)
      ak(3)=a(3)
      ak(4)=0.5
      vp0=0.0
      tr0=0.0
      tcw=0.0
      if(igrad.eq.3)chmlen=cham/areab
      zb=chmlen
      zp=chmlen
      grlen=0.
      grdiam=0.
      egama=0.
      do 5 i=1,nprop
      grlen=grlen+chwp(i)*glenp(i)
      grdiam=grdiam+chwp(i)*gdiap(i)
      ibo(i)=0
      egama=egama+chwp(i)*gamap(i)
```

```
      nsl(i)=0
 5    vp0=chwp(i)/rhop(i)+vp0
      volgi=cham-vp0-chwi*covi
      grlen=grlen/(tmpi-chwi)
      grdiam=grdiam/(tmpi-chwi)
      egama=(egama+chwi*gamai)/tmpi
      ism=0
      odlnr=0.
      vf0=cham-vp0
      eps0=1.-vp0/cham
      eps=eps0
      gasden=chwi/vf0
      prden=tmpi/vp0
      ug=0.
      up=0.
      pmean=forcig*chwi/volgi
      pbase=pmean
      pbrch=pmean
      opbase=pmean
      volg=volgi
      volgi=volgi+vp0
      wallt=twal
      tgas=tempi
      told=0.
      tgaso=tgas
      dtgaso=0.
      cov1=covi
      t=0.
      ptime=0.0
      ibrp=10
      z(3)=1.
      nde=ibrp+nprop
      write(6,132)areab,pmean,vp0,volgi
132   format(1x,'area bore m^2 ',e16.6,/' pressure from ign pa',e16.6,/
     &,' volume of unburnt prop m^3 ',e16.6,/
     &' init cham vol-cov ign m^3 ',e16.6)
      write(6,6)
6     format(1x,'    time         acc           vel          dis             mpress
     &    pbase        pbrch      ')
      isw1=0
19    continue
      do 11 J=1,4
c     FIND BARREL RESISTANCE
      do 201 k=2,npts
      if(y(2)+y(7).ge.trav(k))go to 201
      go to 203
201   continue
      k=npts
203   resp=(trav(k)-y(2)-y(7))/(trav(k)-trav(k-1))
      resp=br(k)-resp*(br(k)-br(k-1))
c     FIND MASS FRACTION BURNED
      do 211 k=1,nprop
      kpr=k
      if(ibo(k).eq.1)goto211
```

```
      nsl1=0
      call prf710(pdp(k),gdiap(k),glenp(k),nperfs(k),y(ibrp+k)
     &,frac(k),volp(k),surf(k),dsdx(k))
      nsl(k)=nsl1
      if(nsl(k).eq.0)goto 212
      if(nslp(k).eq.1)go to 212
      write(6,2213)k
2213  format(' propellant',i2,' has slivered')
      nslp(k)=1
      tsl(k)=y(3)
      ism=1
212   continue
      if(frac(k).lt..9999) go to 211
      frac(k)=1.
      tbo(k)=y(3)
      ibo(k)=1
      ism=1
      write(6,456)k
456   format(' propellant',i2,' has burned out')
211   continue
c     ENERGY LOSS TO PROJECTILE TRANSLATION
      elpt=prwt*y(1)*y(1)/2.
      eptdot=prwt*y(1)*z(1)
c     ENERGY LOSS DUE TO PROJECTILE ROTATION
      elpr=pi*pi*prwt*y(1)*y(1)/4.*twst*twst
      eprdot=pi*pi*prwt*y(1)*z(1)/2.*twst*twst
c     ENERGY LOSS DUE TO GAS AND PROPELLANT MOTION
      if(igrad.eq.1)go to 214
      if(igrad.eq.3)goto 217
      if(igrad.eq.4)go to 438
      pt=y(2)+y(7)
      vzp=bvol+areab*pt
      j4zp=bint(4)+((bvol+areab*pt)**3-bvol**3)/3./areab/areab
      elgpm=tmpi*y(1)*y(1)*areab*areab*j4zp/2./vzp/vzp/vzp
      go to 216
438   pb=y(7)+y(10)
      vzb=bvol+areab*pb
      j4zb=bint(4)+(vzb**3-bvol**3)/3./areab/areab
      elgpm=(1.-eps)*up*up*areab**2*prden*j4zb+
     & eps*ug*ug*areab**2*gasden*j4zb
      elgpm=elgpm/2./vzb/vzb+gasden*areab*ullen/6.*
     & (3.*y(1)*y(1)+3.*y(1)*ullen*dlnrho+ullen**2*dlnrho**2)
c   approximate epdot
      epdot=tmpi*y(1)*z(1)/3.
      go to 216
214   elgpm=tmpi*y(1)*y(1)/6.
      go to 216
217   elgpm=areab*zb/6.*(eps*gasden*ug*ug+(1.-eps)*prden*up*up)
      elgpm=elgpm+gasden*areab*ullen/6.*(3.*y(1)*y(1)+
     & 3.*y(1)*ullen*dlnrho+ullen**2*dlnrho**2)
c   approximate epdot
      epdot=tmpi*y(1)*z(1)/3.
c     ENERGY LOSS FROM BORE RESISTANCE
216   elbr=y(4)
```

```
      z(4)=areab*resp*y(1)
      ebrdot=z(4)
c     ENERGY LOSS DUE TO RECOIL
      elrc=rcwt*y(6)*y(6)/2.
      erdot=rcwt*y(6)*z(6)
c     ENERGY LOSS DUE TO HEAT LOSS
      areaw=cham/areab*pi*bore+2.*areab+pi*bore*(y(2)+y(7))
      avden=0.0
      avc=0.0
      avcp=0.0
      z18=0
      z19=0
      do 213 k=1,nprop
      z18=forcp(k)*gamap(k)*chwp(k)*frac(k)/(gamap(k)-1.)/tempp(k)+z18
      z19=chwp(k)*frac(k)+z19
      avden=avden+chwp(k)*frac(k)
213   continue
      avcp=(z18+forcig*gamai*chwi/(gamai-1.)/tempi)/(z19+chwi)
      avden=(avden+chwi)/(volg+cov1)
      avvel=.5*y(1)
      htns=lambda*avcp*avden*avvel+ho
      z(5)=areaw*htns*(tgas-wallt)*hl
      elht=y(5)
      ehdot=z(5)
      wallt=(elht+htfr*elbr)/cshl/rhocs/areaw/tshl+twal
c     write(6,*)lambda,avcp,avden,avvel,ho,areaw,htns,tgas,wallt,hl,z(5)
c    &,elht
c     ENERGY LOSS DUE TO AIR RESISTANCE
      air=iair
      z(8)=y(1)*pgas*air
      elar=areab*y(8)
      eddot=z(8)*areab
c     RECOIL
      z(6)=0.0
      if(pbrch.le.rp(1)/areab)go to 221
      rfor=rp(2)
      if(y(3)-tr0.ge.tr(2))go to 222
      rfor=(tr(2)-(y(3)-tr0))/(tr(2)-tr(1))
      rfor=rp(2)-rfor*(rp(2)-rp(1))
222   z(6)=areab/rcwt*(pbrch-rfor/areab-resp)
      if(y(6).lt.0.0)y(6)=0.0
      z(7)=y(6)
      goto 223
221   tr0=y(3)
223   continue
c     CALCULATE GAS TEMPERATURE
      eprop=0.0
      rprop=0.0
      dmfogt=0.0
      dmfog=0.0
      do 231 k=1,nprop
      eprop=eprop+forcp(k)*chwp(k)*frac(k)/(gamap(k)-1.)
      rprop=rprop+forcp(k)*chwp(k)*frac(k)/(gamap(k)-1.)/tempp(k)
      dmfogt=dmfogt+forcp(k)*rhop(k)*tng(k)*surf(k)*z(ibrp+k)/
```

```
      & ((gamap(k)-1.)*tempp(k))
      dmfog=dmfog+forcp(k)*rhop(k)*tng(k)*surf(k)*z(ibrp+k)/
      & (gamap(k)-1.)
231   continue
      tenerg=elpt+elpr+elgpm+elbr+elrc+elht+elar
      tgas=(eprop+forcig*chwi/(gamai-1.)-elpt-elpr-elgpm-elbr-elrc-elht
      &-elar)/(rprop+forcig*chwi/(gamai-1.)/tempi)
      tedot=epdot+eprdot+eddot+ebrdot+erdot+ehdot+eptdot
      dtgas=(dmfog-tedot-tgas*dmfogt)/(rprop+forcig*chwi/
      & (gamai-1.)/tempi)
c     FIND FREE VOLUME
      v1=0.0
      cov1=0.0
      do 241 k=1,nprop
      v1=chwp(k)*(1.-frac(k))/rhop(k)+v1
      cov1=cov1+chwp(k)*covp(k)*frac(k)
241   continue
      volg=volgi+areab*(y(2)+y(7))-v1-cov1
c     CALCULATE MEAN PRESSURE
      r1=0.0
      do 251 k=1,nprop
      r1=r1+forcp(k)*chwp(k)*frac(k)/tempp(k)
251   continue
      pmean=tgas/volg*(r1+forcig*chwi/tempi)
259   resp=resp+pgas*air
      if(igrad.eq.1)go to 252
      if(igrad.eq.2)goto 403
      if(igrad.eq.3)go to 404
      if(igrad.eq.4)go to 441
403   if(isw1.ne.0)go to 253
      pbase=pmean
      pbrch=pmean
      if(pbase.gt.resp+1.)isw1=1
      go to 257
c     USE CHAMBRAGE PRESSURE GRADIENT EQUATION
253   j1zp=bint(1)+(bvol*pt+areab/2.*pt*pt)/areab
      j2zp=(bvol+areab*pt)**2/areab/areab
      j3zp=bint(3)+areab*bint(1)*pt+bvol*pt*pt/2.+areab*pt*pt*pt/6.
      a2t=-tmpi*areab*areab/prwt/vzp/vzp
      alf=1.-a2t*j1zp
      a1t=tmpi*areab*(areab*y(1)*y(1)/vzp+areab*resp/prwt)/vzp/vzp
      bt=-tmpi*y(1)*y(1)*areab*areab/2./vzp/vzp/vzp
      bata=-a1t*j1zp-bt*j2zp
      gamma=alf+a2t*j3zp/vzp
      delta=bata+a1t*j3zp/vzp+bt*j4zp/vzp
c     calculate base pressure
      pbase=(pmean-delta)/gamma
c     calculate breech pressure
      pbrch=alf*pbase+bata
      go to 254
c     USE 2 PHASE GRADIENT EQUATION
404   IF(ISW1.NE.0)GOTO 407
      pbase=pmean
      pbrch=pmean
```

```
      if(pbase.gt.resp+1)isw1=1
      go to 257
407   if(isw1.eq.2)go to 411
      vzp=cham+areab*(y(2)+y(7))
      vzb=cham+areab*(y(10)+y(7))
      phi=0.
      phidot=0.
      dmorho=0.
      dmcov=0.
      dmromw=0.
      rmomw=0.
      vfree=vzp-v1
      do 405 k=1,nprop
      rmomw=rmomw+chwp(k)*frac(k)*forcp(k)/tempp(k)
      phi=chwp(k)*frac(k)+phi
      if(ibo(k).eq.1)go to 405
      dmorho=dmorho+tng(k)*surf(k)*z(ibrp+k)
      phidot=rhop(k)*tng(k)*surf(k)*z(ibrp+k)+phidot
      dmcov=rhop(k)*tng(k)*surf(k)*z(ibrp+k)*covp(k)+dmcov
      dmromw=dmromw+rhop(k)*tng(k)*surf(k)*z(ibrp+k)*
     & forcp(k)/tempp(k)
405   continue
      rmomw=rmomw+chwi*forcig/tempi
      gasmas=phi+chwi
      gasden=gasmas/vfree
      phi=(phi+chwi)/tmpi
      if (phi.gt.0.999) then
      isw1=2
      rbm=pbase/pmean
      rbrm=pbrch/pmean
      if(phi.ge.1.)go to 411
      endif
      dmdt=phidot
      phidot=phidot/tmpi
      vdotov=(dmorho+areab*y(1))/vfree
      dlnrho=dmdt/gasmas-vdotov
      dvoldt=dmorho+areab*y(1)-dmcov
c     GET TIME DERIVATIVE OF MEAN PRESSURE
      dpmdt=(dmromw*tgas-pmean*dvoldt+dtgas*rmomw)/volg
      volprp=0.
      effdia=0.
      dmdmdt=0.
      dmdmor=0.
      avelen=0.
      avedia=0.
      do 406 k=1,nprop
      if(ibo(k).eq.1)go to 406
      volprp=volprp+(1.-frac(k))*chwp(k)/rhop(k)
      dmdmdt=dmdmdt+rhop(k)*tng(k)*dsdx(k)*z(ibrp+k)*z(ibrp+k)
      dmdmdt=dmdmdt+rhop(k)*tng(k)*surf(k)*d2xdt2(k)
      dmdmor=dmdmor+(dsdx(k)*z(ibrp+k)**2+surf(k)*d2xdt2(k))*tng(k)
      effdia=effdia+6.*volp(k)/surf(k)*(1.-frac(k))*chwp(k)
406   continue
      c1t=dmdmdt/gasmas-dmdmor/vfree+vdotov**2-(dmdt/gasmas)**2
```

```
      d2lnr=c1t-areab**2*pbase/vfree/prwt
      d2lnr=d2lnr+areab*areab*resp/vfree/prwt
      zp=chmlen+y(2)+y(7)
      zb=chmlen+y(10)+y(7)
      ullen=zp-zb
      cnow=tmpi-gasmas
      vp=y(1)
      effdia=effdia/cnow
      prden=cnow/volprp
      up=y(9)
      phistr=phi-gasden*areab*ullen/tmpi
      ulldot=vp-up
      dphist=phidot-gasden*areab/tmpi*(ulldot+ullen*dlnrho)
      eps=1.-(1.-phi)*tmpi/prden/vzb
      epsdot=phidot*tmpi/prden/vzb+(1.-phi)*tmpi*up*areab/
     & prden/vzb/vzb
      ug=up+(vp+ullen*dlnrho-up)/eps
      alam=(1.5*grlen/grdiam)**.666666667
      alam=(0.5+grlen/grdiam)/alam
      alam=alam**2.17
c     VIS kg/s/m
      vis=.00007
      ren=gasden/vis*effdia*abs(ug-up)
      if(ren.lt.1.)ren=1.
      fsrg=2.5*alam/ren**.081*((1.-eps)/(1.-eps0)*eps0/eps)**.45
      fsc=fsrg*fs0
      phi2=1.-phi-phistr*(1.-eps)/eps
      phi1p=dphist*ug-phidot*up-phistr*epsdot/eps/eps
     & *(vp+ullen*dlnrho-up)+phistr*ulldot*dlnrho/eps
     & +2.*phistr*ug/zb*(ug-up)
      phi1p=phi1p+phi2*gasden/effdia/prden*(ug-up)**2*fsc
      ak2=1./(1.-phi2*tmpi/prden/vzb)
      phi1=phi1p+phistr*z(1)/eps+ullen*phistr*d2lnr/eps
c     ACCELERATION OF FORWARD BOUNDARY OF PROPELLANT BED
      z(9)=gasden*(ug-up)**2*fsc/prden/effdia+tmpi*phi1*ak2
     &/vzb/prden
      z(10)=y(9)
      e=phistr/eps*(1.-ullen*areab/vfree)*areab/prwt
      dd=ullen*phistr*c1t/eps
      ak11=tmpi*e*ak2/zb/vzb
      ak12=tmpi*ak2*(phi1p+dd)/zb/vzb-ak11*resp
      pbase=pmean-ak12*zb*zb/2.+gasden*ullen*resp*areab/prwt
      pbase=pbase+ak12*zb*zb*(zb/3.+ullen)/2./zp
      pbase=pbase-gasden*ullen**2*areab*resp/2./zp/prwt
      pbase=pbase-gasden*ullen**2/2.*(1.-2.*ullen/3./zp)*
     & (c1t-dlnrho**2)
      pbase=pbase-areab**2*gasden*ullen**2*
     &(1.-2.*ullen/3./zp)*resp/prwt/vfree/2.
      deno=-ak11*zb**3/6./zp-ullen*ak11*zb*zb/2./zp
      deno=deno+gasden*ullen*areab/prwt-areab**2*gasden*ullen**2
     &*(1.-2.*ullen/3./zp)/2./vfree/prwt
      deno=deno-gasden*ullen**2*areab/2./zp/prwt+1.+ak11*zb*zb/2.
      pbase=pbase/deno
      if(ism.eq.0)goto453
```

```
      if(ism.eq.1)goto451
      goto452
451   ism=2
      tss=sqrt(egama*rmomw/gasmas*tgas)
      write(6,*)tss
      tss=ullen/(ullen*odlnr+tss)
      tso=y(3)
      write(6,*)tss,tso
452   coefbp=(tss+tso-y(3)-deltat)/tss
      if(coefbp.gt.1.)coefbp=1.
      if(coefbp.le.0.)then
      coefbp=0.
      ism=0
      endif
      pbase=coefbp*opbase+(1.-coefbp)*pbase
      write(6,*)coefbp,opbase,pbase,ism
453   odlnr=dlnrho
      opbase=pbase
      pbrch=pbase*(1.+ak11*zb*zb/2.+gasden*ullen*areab/prwt
     & -areab**2*gasden*ullen**2/2./vfree/prwt)
      pbrch=pbrch+ak12*zb*zb/2.-gasden*ullen*areab*resp/prwt
      pbrch=pbrch+gasden*ullen**2/2.*(c1t-dlnrho**2)
      pbrch=pbrch+areab**2*gasden*ullen**2*resp/2./vfree/prwt
      go to 254
C     USING RGA GRADIENT
441   if(isw1.ne.0)go to 444
      pbase=pmean
      pbrch=pmean
      if(pbase.gt.resp+1.)isw1=1
      go to 257
444   if(isw1.eq.2)go to 411
      vzp=cham+areab*(y(2)+y(7))
      vzb=cham+areab*(y(10)+y(7))
      j1zb=bint(1)+(bvol*pb+areab/2.*pb*pb)/areab
      j2zb=(bvol+areab*pb)**2/areab/areab
      j3zb=bint(3)+areab*bint(1)*pb+bvol*pb*pb/2.+areab/6.*pb**3
      phi=0.
      phidot=0.
      dmorho=0.
      dmcov=0.
      dmromw=0.
      rmomw=0.
      vfree=vzp-v1
      do 442 k=1,nprop
      rmomw=rmomw+chwp(k)*frac(k)*forcp(k)/tempp(k)
      phi=chwp(k)*frac(k)+phi
      if(ibo(k).eq.1)go to 442
      dmorho=dmorho+tng(k)*surf(k)*z(ibrp+k)
      phidot=rhop(k)*tng(k)*surf(k)*z(ibrp+k)+phidot
      dmcov=rhop(k)*tng(k)*surf(k)*z(ibrp+k)*covp(k)+dmcov
      dmromw=dmromw+rhop(k)*tng(k)*surf(k)*z(ibrp+k)*
     & forcp(k)/tempp(k)
442   continue
      rmomw=rmomw+chwi*forcig/tempi
```

```
      gasmas=phi+chwi
      gasden=gasmas/vfree
      phi=(phi+chwi)/tmpi
      if (phi.gt.0.99) then
      isw1=2
      rbm=pbase/pmean
      rbrm=pbrch/pmean
      if(phi.ge.1.)go o 411
      endif
      dmdt=phidot
      phidot=phidot/tmpi
      vdotov=(dmorho+areab*y(1))/vfree
      dlnrho=dmdt/gasmas-vdotov
      dvoldt=dmorho+areab*y(1)-dmcov
c     get time derivative of mean pressure
      dpmdt=(dmromw*tgas-pmean*dvoldt+dtgas*rmomw)/volg
      volprp=0.
      effdia=0.
      dmdmdt=0.
      dmdmor=0.
      avelen=0.
      avedia=0.
      do 443 k=1,nprop
      if(ibo(k).eq.1)go to 443
      volprp=volprp+(1.-frac(k))*chwp(k)/rhop(k)
      dmdmdt=dmdmdt+rhop(k)*tng(k)*dsdx(k)*z(ibrp+k)*z(ibrp+k)
      dmdmdt=dmdmdt+rhop(k)*tng(k)*surf(k)*d2xdt2(k)
      dmdmor=dmdmor+(dsdx(k)*z(ibrp+k)**2+surf(k)*d2xdt2(k))*tng(k)
      effdia=effdia+6.*volp(k)/surf(k)*(1.-frac(k))*chwp(k)
 443  continue
      c1t=dmdmdt/gasmas-dmdmor/vfree+vdotov**2-(dmdt/gasmas)**2
      d2lnr=c1t-areab**2*pbase/vfree/prwt
      d2lnr=d2lnr+areab*areab*resp/vfree/prwt
      zp=chmlen+y(2)+y(7)
      zb=chmlen+y(10)+y(7)
      ullen=zp-zb
      cnow=tmpi-gasmas
      vp=y(1)
      effdia=effdia/cnow
      prden=cnow/volprp
      up=y(9)
      phistr=phi-gasden*areab*ullen/tmpi
      ulldot=vp-up
      dphist=phidot-gasden*areab/tmpi*(ulldot+ullen*dlnrho)
      eps=1.-(1.-phi)*tmpi/prden/vzb
      epsdot=phidot*tmpi/prden/vzb+(1.-phi)*tmpi*up*areab/
     & prden/vzb/vzb
      ug=up+(vp+ullen*dlnrho-up)/eps
      alam=(1.5*grlen/grdiam)**.666666667
      alam=(0.5+grlen/grdiam)/alam
      alam=alam**2.17
c     VIS kg/s/m
      vis=.00007
      ren=gasden/vis*effdia*abs(ug-up)
```

```
      if(ren.lt.1.)ren=1.
      fsrg=2.5*alam/ren**.081*((1.-eps)/(1.-eps0)*eps0/eps)**.45
      fsc=fsrg*fs0
      phi2=1.-phi-phistr*(1.-eps)/eps
      phi1p=dphist*ug-phidot*up-phistr*epsdot/eps/eps
     & *(vp+ullen*dlnrho-up)+phistr*ulldot*dlnrho/eps
     & +2.*areab*phistr*ug/vzb*(ug-up)
      phi1p=phi1p+phi2*gasden/effdia/prden*(ug-up)**2*fsc
      ak2=1./(1.-phi2*tmpi/prden/vzb)
      phi1=phi1p+phistr*z(1)/eps+ullen*phistr*d2lnr/eps
c     ACCELERATION OF FORWARD BOUNDARY OF PROPELLANT BED
      z(9)=gasden*(ug-up)**2*fsc/prden/effdia+tmpi*phi1*ak2
     &/vzb/prden
      z(10)=y(9)
      phi3=phistr*ug*ug+(1.-phi)*up*up
      e=1.-ullen*areab/vfree
      dd=ullen*phistr*c1t/eps
      a1t=tmpi*areab/vzb/vzb*(phi3*areab/vzb-(phi1p+dd-e
     & *phistr*areab*resp/eps/prwt)*ak2)
      a2t=(-tmpi*e*phistr*areab**2/vzb/vzb/eps/prwt)*ak2
      bt=-tmpi*phi3*areab**2/2./vzb/vzb/vzb
      pbase=pmean-gasden*ullen**2/2.*(c1t-dlnrho**2
     $ +areab**2*resp/prwt/vfree)*(1.-2.*areab*ullen/3./vzp)
      pbase=pbase-a1t*j3zb/vzp-bt*j4zb/vzp-areab*ullen*a1t*j1zb/vzp
      pbase=pbase-areab*bt*ullen*j2zb/vzp-gasden*areab**2*ullen**2
     &*resp/2./vzp/prwt+a1t*j1zb+bt*j2zb+areab*gasden*ullen*resp/prwt
      deno=1.+areab*ullen*a2t*j1zb/vzp-gasden*areab**2*ullen**2
     & /2./vzp/prwt+a2t*j3zb/vzp-a2t*j1zb+gasden*ullen*areab/prwt
      deno=deno-gasden*ullen**2*areab**2/2./vfree/prwt
     & +gasden*areab**3*ullen**3/3./vzp/vfree/prwt
      pbase=pbase/deno
      pbrch=pbase*(1.-a2t*j1zb+gasden*ullen*areab/prwt
     & -gasden*ullen**2*areab**2/2./vfree/prwt)
     &+gasden*ullen**2/2.*(c1t-dlnrho**2+areab**2*resp/prwt/vfree)
     & -a1t*j1zb-bt*j2zb-areab*gasden*ullen*resp/prwt
      go to 254
411   pbase=rbm*pmean
      pbrch=rbrm*pmean
      go to 254
c     USE LAGRANGE PRESSURE GRADIENT EQUATION
252   if(isw1.ne.0)go to 256
      if(pmean.lt.resp)resp=pmean
c     CALCULATE BASE PRESSURE
256   pbase=(pmean+tmpi*resp/3./prwt)/(1.+tmpi/3./prwt)
      if(pbase.gt.resp+1.)isw1=1
c     CALCULATE BREECH PESSURE
      pbrch=pbase+tmpi/2./prwt*(pbase-resp)
c     CALCULATE PROJECTILE ACCELERATION
254   z(1)=areab*(pbase-resp)/prwt
      if(z(1).lt.0.0)go to 257
      go to 258
257   if(isw1.eq.0)z(1)=0.0
258   if(y(1).lt.0.0)y(1)=0.0
      z(2)=y(1)
```

```
c     GET BURNING RATE
      do 264 m=1,nprop
      z(ibrp+m)=0.0
      d2xdt2(m)=0.0
      if(ibo(m).eq.1) goto 264
      do 262 k=1,nbr(m)
      if(pmean.gt.pres(m,k))go to 262
      go to 263
262   continue
      k=nbr(m)
263   pmix=pmean
      if(igrad.eq.3)pmix=pbrch-(ak11*pbase+ak12)/6.*zb*zb
      if(igrad.eq.4)pmix=pbrch+(a1t+a2t*pbase)*j3zb/vzb+bt*j4zb/vzb
      if(pmix.lt..99*pmean)pmix=pmean
      z(ibrp+m)=beta(m,k)*(pmix*1.e-6)**alpha(m,k)
      abr(m)=alpha(m,k)
      bbr(m)=beta(m,k)
      d2xdt2(m)=beta(m,k)*alpha(m,k)*(pmix*1.e-6)
     & **(alpha(m,k)-1.)*dpmdt*1.e-6
264   continue
      do 21 i=1,nde
      d(i)=(z(i)-b(j)*p(i))*a(j)
      y(i)=deltat*d(i)+y(i)
      p(i)=3.*d(i)-ak(j)*z(i)+p(i)
 21   continue
 11   continue
      t=t+deltat
      told=y(3)
      if(pmaxm.gt.pmean)go to 281
      pmaxm=pmean
      tpmaxm=y(3)
281   if(pmaxba.gt.pbase)go to 282
      pmaxba=pbase
      tpmxba=y(3)
282   if(pmaxbr.gt.pbrch)go to 283
      pmaxbr=pbrch
      tpmxbr=y(3)
283   continue
      if(y(3).lt.ptime)go to 272
      ptime=ptime+deltap
      pjt=y(2)+y(7)
      write(6,7)y(3),z(1),y(1),pjt,pmean,pbase,pbrch
7     format(1x,7e11.4)
      if(igrad.gt.2)then
      pjt=y(2)+y(7)
      prt=y(10)+y(7)
      write(6,427)prt,pjt
427   format(1x,'prop travel',e11.4,'proj travel',e11.4)
      endif
272   continue
      if(t.gt.tstop)goto 200
      if(y(2)+y(7).gt.travp)go to 200
      rmvelo=y(1)
      tmvelo=y(3)
```

```
      disto=y(2)+y(7)
      go to 19
200   write(6,311)t,y(3)
311   format(1x,' deltat t', e14.6, ' intg t',e14.6)
      write(6,312)pmaxm,tpmaxm
312   format(1x,'PMAXMEAN Pa ',e14.6,' time at PMAXMEAN sec ',e14.6)
      write(6,313)pmaxba,tpmxba
313   format(1x,'PMAXBASE Pa ',e14.6,' time at PMAXBASE sec ',e14.6)
      write(6,314)pmaxbr,tpmxbr
314   format(1x,'PMAXBREECH Pa ',e14.6,' time at PMAXBREECH sec ',e14.6)
      if(y(2)+y(7).le.travp)go to 303
      dfract=(travp-disto)/(y(2)+y(7)-disto)
      rmvel=(y(1)-rmvelo)*dfract+rmvelo
      tmvel=(y(3)-tmvelo)*dfract+tmvelo
      write(6,318)rmvel,tmvel
318   format(1x,'muzzle VELOCITY m/s ',e14.6,' time of muzzle velocity s
     &ec ',e14.6)
      goto 319
303   write(6,327)y(1),y(3)
327   format(1x,'velocity of projectile m/s ',e14.6,' at this time msec
     &',e14.6)
319   efi=chwi*forcig/(gamai-1.)
      efp=0.0
      do 315 i=1,nprop
      efp=efp+chwp(i)*forcp(i)/(gamap(i)-1.0)
315   continue
      tenerg=efi+efp
      write(6,317)tenerg
317   format(1x,'total initial energy available J = ',e14.6)
      tengas=chwi*forcig*tgas/(gamai-1.)/tempi
      do 135 i=1,nprop
      tengas=(frac(i)*chwp(i)*forcp(i)*tgas/tempp(i)/(gamap(i)-1.))+teng
     &as
      write(6,328)i,frac(i),tbo(i)
328   format(' FOR PROPELLANT ',I2,' MASSFRACT BURNT IS ',e14.6
     &,' at time in sec ',e14.6)
135   continue
      write(6,136)tengas
136   format(1x,'total energy remaining in gas J= ',e14.6)
      write(6,320)elpt
320   format(1x,'energy loss from projectile translation J= ',e14.6)
      write(6,321)elpr
321   format(1x,'energy loss from projectile rotation J= ',e14.6)
      write(6,322)elgpm
322   format(1x,'energy lost to gas and propellant motion J= ',e14.6)
      write(6,323)elbr
323   format(1x,'energy lost to bore resistance J= ',e14.6)
      write(6,324)elrc
324   format(1x,'energy lost to recoil J= ',e14.6)
      write(6,325)elht
325   format(1x,'energy loss from heat transfer J= ',e14.6)
      write(6,326)elar
326   format(1x,'energy lost to air resistance J= ',e14.6)
      stop
```

```
 20   write(*,140)
140   format(1x,'end of file encounter')
      stop
 30   write(*,150)
999   continue
998   continue
150   format(1x,'read or write error')
      stop
      end
      subroutine prf710(pd,gd,gl,np,x,frac,vol,surf,dsdx)
      common nsl,kpr,fracsl(10),dsdxsl(10),surfsl(10),
     & nslp(10),tsl(10),pbrch,pbase,pmean,bbr(10),abr(10),
     & deltat,yar(20) ;grad
       dimension ts(10),coef(10)
       pi=3.141593
      nsl=0
C
C      pd=perforation diameter
C      gd=OUTER DIA
C      gl=GRAIN LENGTH
C      NP=NUMBER OF PERFS
C
C      SURF=OUTPUT SURFACE AREA
C      frac=OUTPUT MASS FRACTION OF PROPELLANT BURNED
C
C      w = web = distance between perforation edges
C        = distance between outside perf edge and edge of grain
C
C      p = distance between perforation centers
C
C      x1 = distance to inner sliver burnout
C
C      x2 = distance to outer sliver burnout (frac=1)
C
      if(np.eq.0) go to 2000
      IF(NP.EQ.1)GO TO 3000
      IF(NF.eq.7)GO TO 61
      if(np.eq.19)go to 4000
      if(np.eq.15)go to 5000
   60 WRITE(6,90)
   90 FORMAT(1X,'UNACCEPTABLE GRANULATION')
      STOP
 61    w=(gd-3.*pd)/4.
       d=w+pd
       sqr3=sqrt(3.)
       x1=d/sqr3-pd/2.
       x2=(14.-3.*sqr3)*d/13.-pd/2.
       v0=pi/4.*gl*(gd*gd-7.*pd*pd)
       s0=2.*v0/gl+pi*gl*(gd+7.*pd)
       if (x.gt.w/2.+.0000001) goto 20
       vol=pi/4.*(gl-2.*x)*((gd-2.*x)**2-7.*(pd+2.*x)**2)
       surf=2.*vol/(gl-2.*x)+pi*(gl-2.*x)*((gd-2.*x)+
     & 7.*(pd+2.*x))
       frac=1.-vol/v0
```

```
      dsdx=-4*pi*(gd+7.*pd-3.*gl+18.*x)
      dsdxsl(kpr)=dsdx
      fracsl(kpr)=frac
      surfsl(kpr)=surf
      return
 20   nsl=1
      coef(kpr)=0.
      if(igrad.eq.1.or.igrad.eq.2)go to 726
      if(nslp(kpr).eq.1)goto 26
      tsl(kpr)=yar(3)
      ts(kpr)=w/2.*(-1.+(pbrch/pmean)**abr(kpr))/
     & (bbr(kpr)*(pbase*1.e-6)**abr(kpr))
 26   continue
      coef(kpr)=(ts(kpr)+tsl(kpr)-(deltat+yar(3)))/ts(kpr)
      if(coef(kpr).gt.1.)coef(kpr)=1.
      if(coef(kpr).lt.0.)coef(kpr)=0.
 726  if(x.ge.x2)goto 30
      s1=0.
      s2=0.
      v1=0.
      v2=0.
      ds1dx=0.
      ds2dx=0.
      y=sqrt((pd+2.*x)**2-d*d)
      theta=atan(y/d)
      a1=theta/4.*(pd+2.*x)**2-d/4.*y
      if(x.ge.x1)goto 25
      v1=3./4.*(gl-2.*x)
      v1=v1*(2.*sqr3*d*d-pi*(pd+2.*x)**2+24.*a1)
      s1=2.*v1/(gl-2.*x)
      s1=s1+3.*(gl-2.*x)*(pi-6.*theta)*(pd+2.*x)
 25   y1=sqrt((gd-2.*x)**2-(5.*d-2.*(pd+2.*x))**2)
      chi=atan(y1/(5.*d-2.*(pd+2.*x)))
      y2=sqrt((pd+2.*x)**2-(3.*d-2.*(pd+2.*x))**2)
      phi=atan(y2/(3.*d-2.*(pd+2.*x)))
      a2=phi*(pd+2.*x)**2-chi*(gd-2.*x)**2
      a2=(a2+2.*sqr3*d*sqrt((3.*d-pd-2.*x)*(3.*d-gd+2.*x)))/8.
      v2=pi*(gd-2.*x)**2-6.*sqr3*d*d-4.*pi*(pd+2.*x)**2
      v2=(v2+24.*(a1+2.*a2))*(gl-2.*x)/4.
      s2=2.*v2/(gl-2.*x)
      s2=s2+(gl-2.*x)*((pi-6.*chi)*(gd-2.*x)+2.*(2.*pi-3.*phi-3.*theta
     & )*(pd+2.*x))
      vol=v1+v2
      surf=s1+s2
      frac=1.-vol/v0
      dsdx=-surf/(x2-x)
      dsdx=coef(kpr)*dsdxsl(kpr)+(1.-coef(kpr))*dsdx
      dsdxsl(kpr)=dsdx
      frac=coef(kpr)*fracsl(kpr)+(1.-coef(kpr))*frac
      fracsl(kpr)=frac
      surf=coef(kpr)*surfsl(kpr)+(1.-coef(kpr))*surf
      surfsl(kpr)=surf
      return
 30   vol=0.
```

```
      surf=0.
      frac=fracsl(kpr)*coef(kpr)+1.-coef(kpr)
      fracsl(kpr)=frac
      if(frac.gt..9999) frac=1.
      if(frac.gt..9999)return
      dsdx=0.
      dsdx=dsdxsl(kpr)*coef(kpr)
      dsdxsl(kpr)=dsdx
      if(abs(dsdx).lt.1.)dsdx=0.
      surf=surfsl(kpr)*coef(kpr)
      surfsl(kpr)=surf
      return
C
C ZERO PERF CALCULATIONS START HERE.
C
 2000  if(gd-2.*x.le.0.0) go to 2001
      v0=pi*gd*gd/4.*gl
      vol=pi*(gd-2.*x)**2/4.*(gl-2.*x)
      frac=1.-vol/v0
      surf=pi/2.*(gd-2.*x)**2+pi*(gd-2.*x)*(gl-2.*x)
      dsdx=-2.*pi*(gd+gl-6.*x)
      return
 2001  surf=0.
      frac=1.0
      vol=0.
      dsdx=0.
      nsl=1
      return
c
c     one perf calculation starts here
c
 3000  if(gd-pd-4.*x.le.0.0) goto 3001
      v0=pi/4.*(gd*gd-pd*pd)*gl
      vol=pi/4.*((gd-2.*x)**2-(pd+2.*x)**2)*(gl-2.*x)
      frac=1.-vol/v0
      surf=pi/2.*((gd-2.*x)**2-(pd+2.*x)**2)
      surf=surf+pi*(gd-2.*x)*(gl-2.*x)
      surf=surf+pi*(pd+2.*x)*(gl-2.*x)
      dsdx=-4.*pi*(gd+pd)
      return
3001   surf=0.
      frac=1.0
      vol=0.
      dsdx=0.
      nsl=1
      return
C
C Below is the calculation for the cylindrical 19 perf grain.
C
C   INPUT
C
C     P = PERF DIAMETER
C     D = GRAIN DIAMETER
C     GL = GRAIN LENGTH
```

```
C     X = DISTANCE BURNT
C
C   OUTPUT
C
C     VOL = THE VOLUME OF ONE GRAIN AT X.
C     SURF = THE SURFACE AREA OF ONE GRAIN AT X.
C     FRAC = THE FRACTION OF GRAIN BURNT AT X.
C
C     W=WEB
C
 4000 p=pd
      d=gd
      W=(D-5.*P)/6.
      PI=3.141592654
      SQRT3=SQRT(3.)
      SQRT5=SQRT(5.)
      SQRT6=SQRT(6.)
      SQRT10=SQRT(10.)
C
C      INITIAL VOLUME AND SURFACE AREA
C
      V0=PI/4.*GL*(D*D-19.*P*P)
      S0=2.*V0/GL+PI*GL*(D+19.*P)
C
C      X1 = DISTANCE TO INNER SLIVERR BURNOUT
C      X2 = DISTANCE TO OUTER SLIVER BURNOUT
C      DBC = DISTANCE BETWEEN PERFORATION CENTERS
C      ASSUMES BURNOUT DOES NOT OCCUR IN LONGITUDINAL DIRECTION
C      W1 = SECONDARY WEB
C
      DBC=W+P
      W1=0.5*(D-P-2.*SQRT3*DBC)
      X1=DBC/SQRT3-P/2.
      X2=0.25*(DBC*(6.-SQRT10)-2.*P)
      IF(X.GT.W/2.)GO TO 110
C
C      NOT SLIVERED YET
C
      VOL=PI/4.*(GL-2.*X)*((D-2.*X)**2-19.*(P+2.*X)**2)
      SURF=2.*VOL/(GL-2.*X)+PI*(GL-2.*X)*(D-2.*X+19.*(P+2.*X))
      dsdx=pi*(-4*D+36*GL-76*P-216*x)
      FRAC=1.-VOL/V0
       dsdxsl(kpr)=dsdx
       fracsl(kpr)=frac
       surfsl(kpr)=surf
      RETURN
C
C      V1=TOTAL VOLUME OF INNER SLIVER, V2=TOTAL VOLUME OF OUTER SLIVER
C      S1=TOTAL SURFACE AREA OF INNER SLIVERS, S2=TOTAL SURFACE AREA OF
C          OUTER SLIVERS
C
 110   V1=0.
      V2=0.
      S1=0.
```

```
      S2=0.
      DELTA=0.
      CHI=0.
      NSL=1
       coef(kpr)=0.
       if(igrad.eq.1.or.igrad.eq.2)go to 727
       if(nslp(kpr).eq.1)goto 728
       tsl(kpr)=yar(3)
       ts(kpr)=w/2.*(-1.+(pbrch/pmean)**abr(kpr))/
     & (bbr(kpr)*(pbase*1.e-6)**abr(kpr))
728    continue
       coef(kpr)=(ts(kpr)+tsl(kpr)-(deltat+yar(3)))/ts(kpr)
       if(coef(kpr).gt.1.)coef(kpr)=1.
       if(coef(kpr).lt.0.)coef(kpr)=0.
727    A3=0.
      IF(X.GE.X2)GO TO 130
      THETA=ACOS(DBC/(P+2.*X))
      A1=THETA/4.*(P+2.*X)**2-DBC/4.*SQRT((P+2.*X)**2-DBC*DBC)
      IF(X.GT.X1)GO TO 120
      V1=3.*(GL-2.*X)*(2.*SQRT3*DBC*DBC-PI*(P+2.*X)**2+24*A1)
      S1=2.*V1/(GL-2.*X)+12.*(GL-2.*X)*(PI-6.*THETA)*(P+2.*X)
120    PHI=ACOS((5.*D-13.*P-36.*X)/(12.*(P+2.*X)))
      XI=ACOS((13.*D-5.*P-36.*X)/(12.*(D-2.*X)))
      IF(X.LE.W1/2.)GO TO 125
      DELTA=ACOS((2.*D-P-6.*X)/(SQRT3*(D-2.*X)))
      CHI=ACOS((D-2.*P-6.*X)/(SQRT3*(P+2.*X)))
      A3=.125*(CHI*(P+2.*X)**2-DELTA*(D-2.*X)**2
     & +2.*SQRT6*DBC*SQRT(6.*DBC*(P+2.*X-DBC)-(P+2.*X)**2))
125    A2=.125*(PHI*(P+2.*X)**2-XI*(D-2.*X)**2
     & +2.*SQRT5*DBC*SQRT((5.*DBC-P-2.*X)*(5.*DBC-D+2.*X)))
       V2=.25*(GL-2.*X)*(PI*(D-2.*X)**2-7.*PI*(P+2.*X)**2
     &  -24.*SQRT3*DBC*DBC+48.*(A1+A2+A3))
       S2=2.*V2/(GL-2.*X)+(GL-2.*X)*((D-2.*X)*(PI-6.*(XI+DELTA))
     & +(P+2.*X)*(7.*PI-6.*(2.*THETA+CHI+PHI)))
       VOL=V1+V2
       SURF=S1+S2
       DSDX=-SURF/(X2-X)
       FRAC=1.-VOL/V0
        dsdx=coef(kpr)*dsdxsl(kpr)+(1.-coef(kpr))*dsdx
        dsdxsl(kpr)=dsdx
        frac=coef(kpr)*fracsl(kpr)+(1.-coef(kpr))*frac
        fracsl(kpr)=frac
        surf=coef(kpr)*surfsl(kpr)+(1.-coef(kpr))*surf
        surfsl(kpr)=surf
       RETURN
130    VOL=0.
       SURF=0.
        frac=fracsl(kpr)*coef(kpr)+1.-coef(kpr)
        fracsl(kpr)=frac
        if(frac.gt..9999) frac=1.
        if(frac.gt..9999)return
        dsdx=0.
        dsdx=dsdxsl(kpr)*coef(kpr)
        dsdxsl(kpr)=dsdx
```

```
      if(abs(dsdx).lt.1.)dsdx=0.
      surf=surfsl(kpr)*coef(kpr)
      surfsl(kpr)=surf
     RETURN
C
C Below is the calculation for the 19 perf hex grain.
C
C
C Translation of the input values.
C     p= perf diameter
C     d= grain diameter
C     gl= grain length
C     x= distance burnt
C
C Translation of the output values.
C     vol= volume of one grain at x.
C     surf= surface area of one grain at x.
C     frac= mass fraction of the grain burnt at x.
C
C Assignment statement for pi.
5000  pi=3.141592654
      sqrt3=sqrt(3.)
      p=pd
      d=gd
C
C     d=6w + 5p is the statement for the grain diameter which will be
C used to calculate the web.
C
C To calculate the web.
      w= (d-5.*p)/6.
C
C Below is the equation to calculate the distance between the perf cen-
C ters.
      dpc= p + w
C To calculate the grain diameter between the flats.
      f= 2.*(sqrt3*dpc + p/2. + w)
C
C To calculate the distance burnt.
      X1=dpc/sqrt3-p/2.
      X2= 0.125*(5.*dpc-4.*p)
C
C To calculate the area.
      A=sqrt3/3.*((w+p/2.)**2)-pi/6.*((w+p/2.)**2)
C To calculate the initial volume of the sharp corner grain.
      Vs=gl/4.*(2.*sqrt3*f**2-19.*pi*p**2)
C
C To calculate the volume that will be removed from the grain.
      Vr=6.*A*gl
C
C To calculate the initial volume for the 19hex grain with rounded
C corners.
      Vo= Vs - Vr
C
C To calculate the initial surface area of the sharp corner grain.
```

```
      Ss=2.*Vs/gl+gl*(2.*sqrt3*f+19.*pi*p)
C
C To calculate the surface area that will be removed from the grain.
      Sr=12.*A+gl*(w+p/2.)*(4.*sqrt3-2.*pi)
C
C To calculate the initial surface area for the 19hex grian with rounded
C corners.
      So= Ss-Sr
C
C To calculate the unknows of the grain under the condition x.le.5*w.
      if(0.le.x.and.x.le.w/2.) then
      A=sqrt3/3.*(w-2.*x+(p+2.*x)/2.)**2-pi/6.*
     &(w-2.*x+(p+2.*x)/2.)**2
C To calculate the volume that will be removed from the sharp corner grain.
      Vr=6.*A*(gl-2.*x)
C To calculate the volume for the sharp corner grain at some distance burnt.
      Vn=.25*(gl-2.*x)*(2.*sqrt3*(f-2.*x)**2.
     & -19.*pi*(p+2.*x)**2.)
C
C To calculate the volume for the 19hex grain with rounded corners.
      V= Vn-Vr
C
C To calculate the surface area that will be removed from the sharp
C corner grain.
      Sr=12.*A+(gl-2.*x)*(w-2*x+(p+2.*x)/2.)*(4.*sqrt3-2.*pi)
C To calculate the surface area for the sharp corner grain.
      Sn=2.*V/(gl-2.*x)+(gl-2.*x)*(2.*sqrt3*(f-2.*x)+
     & 19.*pi*(p+2.*x))
C
C To calculate the surface area for 19hex grain with rounded corners.
      S= Sn-Sr
C To calculate the mass fraction.
      frac= 1-V/Vo
      dsdx=-8.*sqrt3*(f-2.*x)-76.*pi*(p+2.*x)+(gl-2.*x)*(-4.*sqrt3+38.
     & *pi)+16*sqrt3*(w+p/2.-x)-8.*pi*(w+p/2.-x)+(gl-2.*x)*
     & (4.*sqrt3-2.*pi)
      surf=S
      vol=V
       dsdxsl(kpr)=dsdx
       fracsl(kpr)=frac
       surfsl(kpr)=surf
      return
      endif
C
C Due to the cross section at the sliver point x=.5*w there will be 24
C identical inner slivers,12 identical side slivers. After slivering the
C surface area and the volume function become more complex. Each type of
C sliver will be treated seperately and later the volumes will be combined
C to complete the function.
C
C To calculate the 12 identical side slivers for the grain x=.5/w.
      nsl=1
       coef(kpr)=0.
       if(igrad.eq.1.or.igrad.eq.2)go to 729
```

```
      if(nslp(kpr).eq.1)goto 730
      tsl(kpr)=yar(3)
      ts(kpr)=w/2.*(-1.+(pbrch/pmean)**abr(kpr))/
     & (bbr(kpr)*(pbase*1.e-6)**abr(kpr))
730   continue
      coef(kpr)=(ts(kpr)+tsl(kpr)-(deltat+yar(3)))/ts(kpr)
      if(coef(kpr).gt.1.)coef(kpr)=1.
      if(coef(kpr).lt.0.)coef(kpr)=0.
729   if(w/2.lt.x.and.x.lt.X1.and.x.lt.X2) then
C
C To calculate the areas of the grain.
      A=sqrt3/3.*(w-2.*x+(p+2.*x)/2.)**2-pi/6.*
     &(w-2.*x+(p+2.*x)/2.)**2
      theta=acos(dpc/(p+2.*x))
      A1=theta/4.*(p+2.*x)**2-dpc/4.*sqrt((p+2.*x)**2-dpc**2)
      omega=acos(2.*dpc/(p+2.*x)-1.)
      A2=0.125*(p+2.*x)*((p+2.*x)*(omega+sin(omega))-2.*dpc*sin(omega))
C To calculate the volumes of the grain.
      V1=3.*(gl-2.*x)*(2.*sqrt3*dpc**2-pi*(p+2.*x)**2+24.*A1)
      V2=6.*(gl-2.*x)*(2.*dpc**2-dpc*(p+2.*x)-pi/4.*(p+2.*x)**2
     &+2.*A1+4.*A2)
C To calculate the surface areas of the grain.
      S1=2.*V1/(gl-2.*x)+12.*(gl-2.*x)*(pi-6.*theta)*(p+2.*x)
      S2=2.*V2/(gl-2.*x)+12.*(gl-2.*x)*(dpc+(p+2.*x)*(pi/2.-omega
     &-theta-sin(omega)))
C To calculate the total volume and total surface area.
      Vf=V1+V2
      Sf=S1+S2
C To calculate the mass fraction.
      frac=1.-Vf/Vo
      surf=Sf
      dsdx=-surf/(x2-x)
      vol=Vf
       dsdx=coef(kpr)*dsdxsl(kpr)+(1.-coef(kpr))*dsdx
       dsdxsl(kpr)=dsdx
       frac=coef(kpr)*fracsl(kpr)+(1.-coef(kpr))*frac
       fracsl(kpr)=frac
       surf=coef(kpr)*surfsl(kpr)+(1.-coef(kpr))*surf
       surfsl(kpr)=surf
      return
      endif
      if(x.gt.X1.and.x.lt.X2)then
C To calculate the area of the grain.
      A=sqrt3/3.*(w-2.*x+(p+2.*x)/2.)**2-pi/6.*
     &(w-2.*x+(p+2.*x)/2.)**2
      theta=acos(dpc/(p+2.*x))
      A1=theta/4.*(p+2.*x)**2-dpc/4.*sqrt((p+2.*x)**2-dpc**2)
      omega=acos(2.*dpc/(p+2.*x)-1.)
      A2=0.125*(p+2.*x)*((p+2.*x)*(omega+sin(omega))-2.*dpc*sin(omega))
C To calculate the volume of the grain.
      V2=6.*(gl-2.*x)*(2.*dpc**2-dpc*(p+2.*x)-pi/4.*(p+2.*x)**2
     &+2.*A1+4.*A2)
C To calculate the surface area of the grain.
      S2=2.*V2/(gl-2.*x)+12.*(gl-2.*x)*(dpc+(p+2.*x)*(pi/2.-omega
```

```
   &-theta-sin(omega)))
C To calculate the volume and the surface area.
      Vf=V2
      Sf=S2
C To calculate the the mass fraction.
      frac=1-Vf/Vo
      surf=Sf
      dsdx=-surf/(x2-x)
      vol=Vf
       dsdx=coef(kpr)*dsdxsl(kpr)+(1.-coef(kpr))*dsdx
       dsdxsl(kpr)=dsdx
       frac=coef(kpr)*fracsl(kpr)+(1.-coef(kpr))*frac
       fracsl(kpr)=frac
       surf=coef(kpr)*surfsl(kpr)+(1.-coef(kpr))*surf
       surfsl(kpr)=surf
      return
      endif
      if(x.gt.X2)then
      dsdx=0.
      surf=0.
      vol=0.
       frac=fracsl(kpr)*coef(kpr)+1.-coef(kpr)
       fracsl(kpr)=frac
       if(frac.gt..9999) frac=1.
       if(frac.gt..9999)return
       dsdx=0.
       dsdx=dsdxsl(kpr)*coef(kpr)
       dsdxsl(kpr)=dsdx
       if(abs(dsdx).lt.1.)dsdx=0.
       surf=surfsl(kpr)*coef(kpr)
       surfsl(kpr)=surf
      return
      endif
      end
```

INTENTIONALLY LEFT BLANK.

APPENDIX 6:
INPUT DATA FOR IBRGA TEST

INTENTIONALLY LEFT BLANK.

```
   9832.2384 12.7 12.7 1.0 0.0  457.2 4 1.00
 3  0. 15.381732 46.482 15.381732 54.102 12.7
   9.796 0 0.0 0.0
 5  0.0 0.0 0.0 .6 0.0 1.3 0.0 300. 0. 457.
   1.e20 2 3.0e+4 0.0 8.0e+5 0.2
   .001135 .01143 .46028 273. 1. 7.8612
   106.34 .9755 294. .004712 1.4
 1
1135.84 3142.75 .97544 9.7959 1.6605 1.23 7 3.175 .0508  1.23063
 1  1.0 .1105187 689.476
   .01 .1 30.
```

INTENTIONALLY LEFT BLANK.

APPENDIX 7:
OUTPUT FROM IBRGA

INTENTIONALLY LEFT BLANK.

the input file is i6315a
using rga gradient

chamber distance cm   chamber diameter cm
0.000000E+00   0.153817E+02
0.464820E+02   0.153817E+02
0.541020E+02   0.127000E+02
chamber volume cm**3 0.982089E+04
groove diam cm 0.127000E+02
land diam cm 0.127000E+02
groove/land ratio 0.100000E+01
twist turns/caliber 0.000000E+00
projectile travel cm 0.457200E+03
gradient # 4
friction factor 0.100000E+01

projectile mass kg 0.979600E+01
switch to calculate energy lost to air resistance J 0
fraction of work against bore used to heat the tube 0.000000E+00
gas pressure Pa 0.000000E+00
number barrel resistance points 5
bore resistance MPa - travel cm
0.000000E+00 0.000000E+00
0.000000E+00 0.600000E+00
0.000000E+00 0.130000E+01
0.000000E+00 0.300000E+03
0.000000E+00 0.457000E+03

mass of recoiling parts kg 0.100000E+21
number of recoil point pairs 2
recoil force N recoil time sec
0.300000E+05   0.000000E+00
0.800000E+06   0.200000E+00

free convective heat transfer coefficient w/cm**2 K 0.113500E-02
chamber wall thickness cm 0.114300E-01
heat capacity of steel of chamber wall J/g K 0.460280E+00
initial temperature of chamber wall K 0.273000E+03
heat loss coefficient 0.100000E+01
density of chamber wall steel g/cm**3 0.786120E+01

impetus of igniter propellant J/g 0.106340E+03
covolume of igniter cm**3/g 0.975500E+00
adiabatic flame temperature of igniter propellant K 0.294000E+03
initial mass of igniter kg 0.471200E-02
ratio of specific heats for igniter 0.140000E+01
there are 1 propellants
for propellant number 1
impetus of propellant J/g 0.113584E+04

adiabatic temperature of propellant K 0.314275E+04
covolume of propellant cm**3/g 0.975440E+00
initial mass of propellant kg 0.979590E+01
density of propellant g/cm**3 0.166050E+01
ratio of specific heats for propellant 0.123000E+01
number of perforations of propellant 7
length of propellant grain cm 0.317500E+01
diameter of perforation in propellant grains cm 0.508000E-01
outside diameter of propellant grain cm 0.123063E+01

for propellant 1 the total number of grains is 0.158099E+04
number of burning rate points 1

exponent	coefficient	pressure
-	cm/sec-MPa**ai	MPa
0.100000E+01	0.110519E+00	0.689476E+03

time increment msec 0.100000E-01 print increment msec 0.100000E+00
time to stop calculation msec 0.300000E+02
area bore m^2 0.126677E-01
pressure from ign pa 0.127925E+06
volume of unburnt prop m^3 0.589937E-02
init cham vol-cov ign m^3 0.981629E-02

```
   time       acc        vel        dis        mpress     pbase      pbrch
 0.1000E-04 0.1696E+03 0.1399E-02 0.5582E-08 0.1312E+06 0.1312E+06 0.1313E+06
prop travel 0.8638E-11proj travel 0.5582E-08
 0.1100E-03 0.2182E+03 0.2070E-01 0.1070E-05 0.1688E+06 0.1688E+06 0.1689E+06
prop travel 0.1936E-08proj travel 0.1070E-05
 0.2100E-03 0.2790E+03 0.4545E-01 0.4327E-05 0.2159E+06 0.2158E+06 0.2160E+06
prop travel 0.8074E-08proj travel 0.4327E-05
 0.3100E-03 0.3543E+03 0.7699E-01 0.1039E-04 0.2742E+06 0.2740E+06 0.2743E+06
prop travel 0.2001E-07proj travel 0.1039E-04
 0.4100E-03 0.4464E+03 0.1169E+00 0.2000E-04 0.3454E+06 0.3452E+06 0.3455E+06
prop travel 0.4001E-07proj travel 0.2000E-04
 0.5100E-03 0.5576E+03 0.1669E+00 0.3410E-04 0.4315E+06 0.4312E+06 0.4317E+06
prop travel 0.7140E-07proj travel 0.3410E-04
 0.6100E-03 0.6904E+03 0.2291E+00 0.5379E-04 0.5344E+06 0.5339E+06 0.5346E+06
prop travel 0.1190E-06proj travel 0.5379E-04
 0.7100E-03 0.8474E+03 0.3058E+00 0.8040E-04 0.6560E+06 0.6553E+06 0.6563E+06
prop travel 0.1898E-06proj travel 0.8040E-04
 0.8100E-03 0.1031E+04 0.3995E+00 0.1155E-03 0.7986E+06 0.7975E+06 0.7990E+06
prop travel 0.2943E-06proj travel 0.1155E-03
 0.9100E-03 0.1245E+04 0.5130E+00 0.1610E-03 0.9646E+06 0.9631E+06 0.9653E+06
prop travel 0.4479E-06proj travel 0.1610E-03
 0.1010E-02 0.1493E+04 0.6497E+00 0.2189E-03 0.1157E+07 0.1155E+07 0.1158E+07
prop travel 0.6734E-06proj travel 0.2189E-03
 0.1110E-02 0.1780E+04 0.8130E+00 0.2918E-03 0.1380E+07 0.1376E+07 0.1381E+07
prop travel 0.1005E-05proj travel 0.2918E-03
 0.1210E-02 0.2110E+04 0.1007E+01 0.3825E-03 0.1637E+07 0.1632E+07 0.1639E+07
prop travel 0.1494E-05proj travel 0.3825E-03
 0.1310E-02 0.2492E+04 0.1237E+01 0.4944E-03 0.1934E+07 0.1927E+07 0.1936E+07
prop travel 0.2217E-05proj travel 0.4944E-03
 0.1410E-02 0.2931E+04 0.1507E+01 0.6312E-03 0.2276E+07 0.2266E+07 0.2281E+07
```

prop travel 0.3292E-05proj travel 0.6312E-03
0.1510E-02 0.3437E+04 0.1825E+01 0.7974E-03 0.2673E+07 0.2658E+07 0.2679E+07
prop travel 0.4893E-05proj travel 0.7974E-03
0.1610E-02 0.4021E+04 0.2197E+01 0.9981E-03 0.3131E+07 0.3110E+07 0.3141E+07
prop travel 0.7289E-05proj travel 0.9981E-03
0.1710E-02 0.4694E+04 0.2632E+01 0.1239E-02 0.3662E+07 0.3630E+07 0.3676E+07
prop travel 0.1089E-04proj travel 0.1239E-02
0.1810E-02 0.5470E+04 0.3140E+01 0.1527E-02 0.4277E+07 0.4230E+07 0.4298E+07
prop travel 0.1630E-04proj travel 0.1527E-02
0.1910E-02 0.6361E+04 0.3730E+01 0.1870E-02 0.4989E+07 0.4919E+07 0.5020E+07
prop travel 0.2447E-04proj travel 0.1870E-02
0.2010E-02 0.7384E+04 0.4416E+01 0.2276E-02 0.5814E+07 0.5710E+07 0.5859E+07
prop travel 0.3681E-04proj travel 0.2276E-02
0.2110E-02 0.8556E+04 0.5212E+01 0.2757E-02 0.6769E+07 0.6616E+07 0.6836E+07
prop travel 0.5544E-04proj travel 0.2757E-02
0.2210E-02 0.9892E+04 0.6133E+01 0.3323E-02 0.7874E+07 0.7650E+07 0.7974E+07
prop travel 0.8353E-04proj travel 0.3323E-02
0.2310E-02 0.1141E+05 0.7196E+01 0.3988E-02 0.9153E+07 0.8825E+07 0.9300E+07
prop travel 0.1258E-03proj travel 0.3988E-02
0.2410E-02 0.1313E+05 0.8421E+01 0.4767E-02 0.1063E+08 0.1015E+08 0.1085E+08
prop travel 0.1891E-03proj travel 0.4767E-02
0.2510E-02 0.1507E+05 0.9829E+01 0.5678E-02 0.1234E+08 0.1165E+08 0.1265E+08
prop travel 0.2835E-03proj travel 0.5678E-02
0.2610E-02 0.1724E+05 0.1144E+02 0.6740E-02 0.1431E+08 0.1333E+08 0.1475E+08
prop travel 0.4232E-03proj travel 0.6740E-02
0.2710E-02 0.1967E+05 0.1328E+02 0.7974E-02 0.1658E+08 0.1521E+08 0.1719E+08
prop travel 0.6278E-03proj travel 0.7974E-02
0.2810E-02 0.2238E+05 0.1538E+02 0.9405E-02 0.1918E+08 0.1731E+08 0.2003E+08
prop travel 0.9242E-03proj travel 0.9405E-02
0.2910E-02 0.2541E+05 0.1777E+02 0.1106E-01 0.2217E+08 0.1965E+08 0.2332E+08
prop travel 0.1348E-02proj travel 0.1106E-01
0.3010E-02 0.2882E+05 0.2047E+02 0.1297E-01 0.2560E+08 0.2229E+08 0.2710E+08
prop travel 0.1943E-02proj travel 0.1297E-01
0.3110E-02 0.3268E+05 0.2354E+02 0.1517E-01 0.2951E+08 0.2527E+08 0.3144E+08
prop travel 0.2765E-02proj travel 0.1517E-01
0.3210E-02 0.3709E+05 0.2702E+02 0.1769E-01 0.3397E+08 0.2869E+08 0.3637E+08
prop travel 0.3877E-02proj travel 0.1769E-01
0.3310E-02 0.4217E+05 0.3098E+02 0.2059E-01 0.3903E+08 0.3261E+08 0.4194E+08
prop travel 0.5351E-02proj travel 0.2059E-01
0.3410E-02 0.4802E+05 0.3548E+02 0.2391E-01 0.4475E+08 0.3714E+08 0.4820E+08
prop travel 0.7263E-02proj travel 0.2391E-01
0.3510E-02 0.5475E+05 0.4060E+02 0.2770E-01 0.5119E+08 0.4234E+08 0.5518E+08
prop travel 0.9691E-02proj travel 0.2770E-01
0.3610E-02 0.6243E+05 0.4644E+02 0.3205E-01 0.5841E+08 0.4828E+08 0.6294E+08
prop travel 0.1271E-01proj travel 0.3205E-01
0.3710E-02 0.7109E+05 0.5310E+02 0.3702E-01 0.6645E+08 0.5497E+08 0.7153E+08
prop travel 0.1641E-01proj travel 0.3702E-01
0.3810E-02 0.8072E+05 0.6068E+02 0.4270E-01 0.7536E+08 0.6242E+08 0.8100E+08
prop travel 0.2085E-01proj travel 0.4270E-01
0.3910E-02 0.9131E+05 0.6926E+02 0.4919E-01 0.8514E+08 0.7061E+08 0.9139E+08
prop travel 0.2611E-01proj travel 0.4919E-01
0.4010E-02 0.1028E+06 0.7895E+02 0.5659E-01 0.9580E+08 0.7948E+08 0.1027E+09
prop travel 0.3228E-01proj travel 0.5659E-01
0.4110E-02 0.1151E+06 0.8983E+02 0.6502E-01 0.1073E+09 0.8898E+08 0.1150E+09

prop travel 0.3944E-01proj travel 0.6502E-01
0.4210E-02 0.1280E+06 0.1020E+03 0.7460E-01 0.1197E+09 0.9901E+08 0.1282E+09
prop travel 0.4767E-01proj travel 0.7460E-01
0.4310E-02 0.1416E+06 0.1154E+03 0.8546E-01 0.1327E+09 0.1095E+09 0.1421E+09
prop travel 0.5707E-01proj travel 0.8546E-01
0.4410E-02 0.1555E+06 0.1303E+03 0.9773E-01 0.1464E+09 0.1203E+09 0.1568E+09
prop travel 0.6774E-01proj travel 0.9773E-01
0.4510E-02 0.1697E+06 0.1465E+03 0.1116E+00 0.1605E+09 0.1313E+09 0.1721E+09
prop travel 0.7980E-01proj travel 0.1116E+00
0.4610E-02 0.1840E+06 0.1642E+03 0.1271E+00 0.1748E+09 0.1423E+09 0.1877E+09
prop travel 0.9336E-01proj travel 0.1271E+00
0.4710E-02 0.1981E+06 0.1833E+03 0.1444E+00 0.1893E+09 0.1532E+09 0.2035E+09
prop travel 0.1085E+00proj travel 0.1444E+00
0.4810E-02 0.2118E+06 0.2038E+03 0.1638E+00 0.2036E+09 0.1638E+09 0.2192E+09
prop travel 0.1254E+00proj travel 0.1638E+00
0.4910E-02 0.2249E+06 0.2256E+03 0.1852E+00 0.2175E+09 0.1739E+09 0.2347E+09
prop travel 0.1442E+00proj travel 0.1852E+00
0.5010E-02 0.2372E+06 0.2487E+03 0.2089E+00 0.2308E+09 0.1834E+09 0.2496E+09
prop travel 0.1648E+00proj travel 0.2089E+00
0.5110E-02 0.2485E+06 0.2730E+03 0.2350E+00 0.2433E+09 0.1922E+09 0.2637E+09
prop travel 0.1876E+00proj travel 0.2350E+00
0.5210E-02 0.2587E+06 0.2983E+03 0.2636E+00 0.2547E+09 0.2001E+09 0.2769E+09
prop travel 0.2124E+00proj travel 0.2636E+00
0.5310E-02 0.2677E+06 0.3247E+03 0.2947E+00 0.2651E+09 0.2070E+09 0.2889E+09
prop travel 0.2395E+00proj travel 0.2947E+00
0.5410E-02 0.2754E+06 0.3518E+03 0.3285E+00 0.2741E+09 0.2129E+09 0.2998E+09
prop travel 0.2689E+00proj travel 0.3285E+00
0.5510E-02 0.2817E+06 0.3797E+03 0.3651E+00 0.2819E+09 0.2178E+09 0.3092E+09
prop travel 0.3006E+00proj travel 0.3651E+00
0.5610E-02 0.2867E+06 0.4081E+03 0.4045E+00 0.2883E+09 0.2217E+09 0.3172E+09
prop travel 0.3348E+00proj travel 0.4045E+00
0.5710E-02 0.2903E+06 0.4370E+03 0.4467E+00 0.2933E+09 0.2245E+09 0.3239E+09
prop travel 0.3714E+00proj travel 0.4467E+00
0.5810E-02 0.2928E+06 0.4661E+03 0.4919E+00 0.2970E+09 0.2264E+09 0.3291E+09
prop travel 0.4105E+00proj travel 0.4919E+00
0.5910E-02 0.2940E+06 0.4955E+03 0.5400E+00 0.2995E+09 0.2274E+09 0.3329E+09
prop travel 0.4520E+00proj travel 0.5400E+00
0.6010E-02 0.2942E+06 0.5249E+03 0.5910E+00 0.3008E+09 0.2275E+09 0.3355E+09
prop travel 0.4961E+00proj travel 0.5910E+00
0.6110E-02 0.2934E+06 0.5543E+03 0.6449E+00 0.3010E+09 0.2269E+09 0.3369E+09
prop travel 0.5428E+00proj travel 0.6449E+00
0.6210E-02 0.2918E+06 0.5835E+03 0.7018E+00 0.3003E+09 0.2257E+09 0.3371E+09
prop travel 0.5919E+00proj travel 0.7018E+00
0.6310E-02 0.2894E+06 0.6126E+03 0.7616E+00 0.2988E+09 0.2238E+09 0.3364E+09
prop travel 0.6435E+00proj travel 0.7616E+00
0.6410E-02 0.2864E+06 0.6414E+03 0.8243E+00 0.2965E+09 0.2215E+09 0.3348E+09
prop travel 0.6976E+00proj travel 0.8243E+00
0.6510E-02 0.2828E+06 0.6699E+03 0.8899E+00 0.2936E+09 0.2187E+09 0.3323E+09
prop travel 0.7542E+00proj travel 0.8899E+00
0.6610E-02 0.2788E+06 0.6979E+03 0.9583E+00 0.2901E+09 0.2156E+09 0.3293E+09
prop travel 0.8132E+00proj travel 0.9583E+00
0.6710E-02 0.2744E+06 0.7256E+03 0.1029E+01 0.2862E+09 0.2122E+09 0.3256E+09
prop travel 0.8746E+00proj travel 0.1029E+01
0.6810E-02 0.2697E+06 0.7528E+03 0.1103E+01 0.2820E+09 0.2085E+09 0.3214E+09

prop travel 0.9384E+00proj travel 0.1103E+01
0.6910E-02 0.2647E+06 0.7795E+03 0.1180E+01 0.2774E+09 0.2047E+09 0.3169E+09
prop travel 0.1005E+01proj travel 0.1180E+01
0.7010E-02 0.2596E+06 0.8058E+03 0.1259E+01 0.2727E+09 0.2008E+09 0.3120E+09
prop travel 0.1073E+01proj travel 0.1259E+01
0.7110E-02 0.2544E+06 0.8315E+03 0.1341E+01 0.2677E+09 0.1968E+09 0.3069E+09
prop travel 0.1144E+01proj travel 0.1341E+01
0.7210E-02 0.2492E+06 0.8566E+03 0.1426E+01 0.2627E+09 0.1927E+09 0.3016E+09
prop travel 0.1217E+01proj travel 0.1426E+01
0.7310E-02 0.2439E+06 0.8813E+03 0.1512E+01 0.2576E+09 0.1886E+09 0.2961E+09
prop travel 0.1292E+01proj travel 0.1512E+01
0.7410E-02 0.2385E+06 0.9054E+03 0.1602E+01 0.2525E+09 0.1845E+09 0.2905E+09
prop travel 0.1369E+01proj travel 0.1602E+01
0.7510E-02 0.2333E+06 0.9290E+03 0.1694E+01 0.2473E+09 0.1804E+09 0.2849E+09
prop travel 0.1449E+01proj travel 0.1694E+01
0.7610E-02 0.2281E+06 0.9521E+03 0.1788E+01 0.2422E+09 0.1764E+09 0.2793E+09
prop travel 0.1530E+01proj travel 0.1788E+01
0.7710E-02 0.2229E+06 0.9746E+03 0.1884E+01 0.2372E+09 0.1724E+09 0.2737E+09
prop travel 0.1614E+01proj travel 0.1884E+01
0.7810E-02 0.2179E+06 0.9967E+03 0.1983E+01 0.2322E+09 0.1685E+09 0.2681E+09
prop travel 0.1700E+01proj travel 0.1983E+01
0.7910E-02 0.2129E+06 0.1018E+04 0.2083E+01 0.2272E+09 0.1646E+09 0.2625E+09
prop travel 0.1787E+01proj travel 0.2083E+01
0.8010E-02 0.2081E+06 0.1039E+04 0.2186E+01 0.2224E+09 0.1609E+09 0.2570E+09
prop travel 0.1877E+01proj travel 0.2186E+01
0.8110E-02 0.2033E+06 0.1060E+04 0.2291E+01 0.2176E+09 0.1572E+09 0.2516E+09
prop travel 0.1969E+01proj travel 0.2291E+01
0.8210E-02 0.1987E+06 0.1080E+04 0.2398E+01 0.2130E+09 0.1537E+09 0.2463E+09
prop travel 0.2062E+01proj travel 0.2398E+01
0.8310E-02 0.1943E+06 0.1100E+04 0.2507E+01 0.2084E+09 0.1502E+09 0.2411E+09
prop travel 0.2158E+01proj travel 0.2507E+01
0.8410E-02 0.1899E+06 0.1119E+04 0.2618E+01 0.2040E+09 0.1469E+09 0.2360E+09
prop travel 0.2255E+01proj travel 0.2618E+01
0.8510E-02 0.1857E+06 0.1138E+04 0.2731E+01 0.1997E+09 0.1436E+09 0.2310E+09
prop travel 0.2354E+01proj travel 0.2731E+01
0.8610E-02 0.1817E+06 0.1156E+04 0.2845E+01 0.1954E+09 0.1405E+09 0.2261E+09
prop travel 0.2455E+01proj travel 0.2845E+01
0.8710E-02 0.1777E+06 0.1174E+04 0.2962E+01 0.1913E+09 0.1374E+09 0.2213E+09
prop travel 0.2558E+01proj travel 0.2962E+01
propellant 1 has slivered
0.8810E-02 0.1772E+06 0.1192E+04 0.3080E+01 0.1852E+09 0.1370E+09 0.2116E+09
prop travel 0.2662E+01proj travel 0.3080E+01
0.8910E-02 0.1773E+06 0.1209E+04 0.3200E+01 0.1814E+09 0.1371E+09 0.2052E+09
prop travel 0.2768E+01proj travel 0.3200E+01
0.9010E-02 0.1726E+06 0.1227E+04 0.3322E+01 0.1765E+09 0.1335E+09 0.1997E+09
prop travel 0.2875E+01proj travel 0.3322E+01
0.9110E-02 0.1674E+06 0.1244E+04 0.3446E+01 0.1714E+09 0.1294E+09 0.1938E+09
prop travel 0.2982E+01proj travel 0.3446E+01
0.9210E-02 0.1621E+06 0.1260E+04 0.3571E+01 0.1662E+09 0.1254E+09 0.1879E+09
prop travel 0.3091E+01proj travel 0.3571E+01
0.9310E-02 0.1570E+06 0.1276E+04 0.3698E+01 0.1610E+09 0.1214E+09 0.1821E+09
prop travel 0.3200E+01proj travel 0.3698E+01
0.9410E-02 0.1519E+06 0.1292E+04 0.3826E+01 0.1559E+09 0.1175E+09 0.1763E+09
prop travel 0.3310E+01proj travel 0.3826E+01

0.9510E-02 0.1470E+06 0.1307E+04 0.3956E+01 0.1509E+09 0.1137E+09 0.1707E+09
prop travel 0.3421E+01proj travel 0.3956E+01
0.9610E-02 0.1422E+06 0.1321E+04 0.4087E+01 0.1461E+09 0.1100E+09 0.1652E+09
prop travel 0.3533E+01proj travel 0.4087E+01
0.9710E-02 0.1376E+06 0.1335E+04 0.4220E+01 0.1414E+09 0.1064E+09 0.1599E+09
prop travel 0.3646E+01proj travel 0.4220E+01
0.9810E-02 0.1332E+06 0.1349E+04 0.4354E+01 0.1368E+09 0.1030E+09 0.1548E+09
prop travel 0.3759E+01proj travel 0.4354E+01
0.9910E-02 0.1289E+06 0.1362E+04 0.4490E+01 0.1325E+09 0.9970E+08 0.1498E+09
prop travel 0.3873E+01proj travel 0.4490E+01
deltat t 0.998000E-02 intg t 0.998000E-02
PMAXMEAN Pa 0.301051E+09 time at PMAXMEAN sec 0.608000E-02
PMAXBASE Pa 0.227560E+09 time at PMAXBASE sec 0.598000E-02
PMAXBREECH Pa 0.337155E+09 time at PMAXBREECH sec 0.618000E-02
muzzle VELOCITY m/s 0.136949E+04 time of muzzle velocity sec 0.997004E-02
total initial energy available J = 0.483777E+08
FOR PROPELLANT 1 MASSFRACT BURNT IS 0.966455E+00 at time in sec 0.000000E+00
total energy remaining in gas J= 0.329127E+08
energy loss from projectile translation J= 0.920305E+07
energy loss from projectile rotation J= 0.000000E+00
energy lost to gas and propellant motion J= 0.311553E+07
energy lost to bore resistance J= 0.000000E+00
energy lost to recoil J= 0.188116E-11
energy loss from heat transfer J= 0.152361E+07
energy lost to air resistance J= 0.000000E+00

APPENDIX 8:

INPUT DATA FOR XKTC

INTENTIONALLY LEFT BLANK.

```
 GRADIENT TESTS 600 IN~ X6315A
TFFFTTT          1 1 0       00
  25   -3    0 3500
10.0       180.0      0.0001     2.0          0.05        0.01         0.0001       0.0001
  300  100 1100  100 1500  100
    6    0    0    5    0    0    1    2    0    0    0    8                        0
    0
529.        14.7        28.896     1.4
529.0
JA2 7PF LOT RH2-5    0.0          21.3         21.6       .06
    7   .4845   0.020        1.25         7.
15000.      1.0         41754.                  .5
 8000.    .00030     1.000      100000.   .000300    1.000       0.0          520.
19826000. 23.0000    1.2300     27.00
0.0         3.0279     18.3          3.0279     21.3         2.5         21.9         2.5
23.37       2.5         210.56      2.5
 6.0        0.          6.25          0.         7.2          00.         7.8            0.
 210.56       0.
7.77         0.0228     0.7
21.3        21.6        44.0        0.000
0.0          3.          12.          30.         40.         50.          80.          30.
```

INTENTIONALLY LEFT BLANK.

No of Copies	Organization
1	Office of the Secretary of Defense OUSD(A) Director, Live Fire Testing ATTN: James F. O'Bryon Washington, DC 20301-3110
2	Administrator Defense Technical Info Center ATTN: DTIC-DDA Cameron Station Alexandria, VA 22304-6145
1	HQDA (SARD-TR) WASH DC 20310-0001
1	Commander US Army Materiel Command ATTN: AMCDRA-ST 5001 Eisenhower Avenue Alexandria, VA 22333-0001
1	Commander US Army Laboratory Command ATTN: AMSLC-DL Adelphi, MD 20783-1145
2	Commander US Army, ARDEC ATTN: SMCAR-IMI-I Picatinny Arsenal, NJ 07806-5000
2	Commander US Army, ARDEC ATTN: SMCAR-TDC Picatinny Arsenal, NJ 07806-5000
1	Director Benet Weapons Laboratory US Army, ARDEC ATTN: SMCAR-CCB-TL Watervliet, NY 12189-4050
1	Commander US Army Armament, Munitions and Chemical Command ATTN: SMCAR-ESP-L Rock Island, IL 61299-5000
1	Commander US Army Aviation Systems Command ATTN: AMSAV-DACL 4300 Goodfellow Blvd. St. Louis, MO 63120-1798

No of Copies	Organization
1	Director US Army Aviation Research and Technology Activity Ames Research Center Moffett Field, CA 94035-1099
1	Commander US Army Missile Command ATTN: AMSMI-RD-CS-R (DOC) Redstone Arsenal, AL 35898-5010
1	Commander US Army Tank-Automotive Command ATTN: AMSTA-TSL (Technical Library) Warren, MI 48397-5000
1	Director US Army TRADOC Analysis Command ATTN: ATAA-SL White Sands Missile Range, NM 88002-5502
(Class. only) 1	Commandant US Army Infantry School ATTN: ATSH-CD (Security Mgr.) Fort Benning, GA 31905-5660
(Unclass. only) 1	Commandant US Army Infantry School ATTN: ATSH-CD-CSO-OR Fort Benning, GA 31905-5660
1	Air Force Armament Laboratory ATTN: AFATL/DLODL Eglin AFB, FL 32542-5000

Aberdeen Proving Ground

No of Copies	Organization
2	Dir, USAMSAA ATTN: AMXSY-D AMXSY-MP, H. Cohen
1	Cdr, USATECOM ATTN: AMSTE-TD
3	Cdr, CRDEC, AMCCOM ATTN: SMCCR-RSP-A SMCCR-MU SMCCR-MSI
1	Dir, VLAMO ATTN: AMSLC-VL-D

No. of Copies	Organization
1	Commander USA Concepts Analysis Agency ATTN: D. Hardison 8120 Woodmont Avenue Bethesda, MD 20014-2797
1	C.I.A. 01R/DB/Standard Washington, DC 20505
1	US Army Ballistic Missile Defense Systems Command Advanced Technology Center P.O. Box 1500 Huntsville, AL 35807-3801
1	Chairman DoD Explosives Safety Board Room 856-C Hoffman Bldg. 1 2461 Eisenhower Avenue Alexandria, VA 22331-0600
1	Commander US Army Material Command ATTN: AMCPM-GCM-WF 5001 Eisenhower Avenue Alexandria, VA 22333-5001
1	Commander US Army Material Command ATTN; AMCDE-DW 5001 Eisenhower Avenue Alexandria, VA 22333-5001
4	Project Manager Autonomous Precision-Guided Munition (APGM) Armament RD&E Center US Army AMCCOM ATTN: AMCPM-CW AMCPM-CWW AMCPM-CWS, M. Fisette AMCPM-CWA-S, R. DeKleine Picatinny Arsenal, NJ 07806-5000
2	Project Manager Production Base Modernization Agency ATTN: AMSMC-PBM, A. Siklosi AMSMC-PBM-E, L. Laibson Picatinny Arsenal, NJ 07806-5000

No. of Copies	Organization
3	PEO-Armaments Project Manger Tank Main Armament Systems ATTN: AMCPM-TMA, K. Russell AMCPM-TMA-105 AMCPM-TMA-120 Picatinny Arsenal, NJ 07806-5000
1	Commander Armament RD&E Center US Army AMCCOM ATTN: SMCAR-AEE Picatinny Arsenal, NJ 07806-5000
8	Commander Armament RD&E Center US Army AMCCOM ATTN: SMCAR-AEE-B A. Beardell B. Brodman D. Downs S. Einstein S. Westley S. Bernstein C. Roller J. Rutkowski Picatinny Arsenal, NJ 07806-5000
2	Commander US Army ARDEC ATTN: SMCAR-AES, S. Kaplowitz; D. Spring Picatinny Arsenal, NJ 07806-5000
2	Commander Armament RD&E Center US Army AMCCOM ATTN: SMCAR-HFM, E. Barrieres SMCAR-CCH-V, C. Mandala Picatinny Arsenal, NJ 07806-5000
1	Commander Armament RD&E Center US Army AMCCOM ATTN: SMCAR-FSA-T, M. Salsbury Picatinny Arsenal, NJ 07806-5000
1	Commander, USACECOM CECOM R&D Technical Library ATTN: ASQNC-ELC-I-T, Myer Center Fort Monmouth, NJ 07703-5301

No. of Copies	Organization
1	Commander US Army Harry Diamond Laboratories ATTN: SLCHD-TA-L 2800 Powder Mill Rd Adelphi, MD 20783-1145
1	Commandant US Army Aviation School ATTN: Aviation Agency Fort Rucker, AL 36360
1	Project Manager US Army Tank-Automotive Command Improved TOW Vehicle ATTN: AMCPM-ITV Warren, MI 48397-5000
2	Program Manager US Army Tank-Automotive Command ATTN: AMCPM-ABMS, T. Dean Warren, MI 48092-2498
1	Project Manager US Army Tank-Automotive Command Fighting Vehicle Systems ATTN: AMCPM-BFVS Warren, MI 48092-2498
1	President US Army Armor and Engineer Board ATTN: ATZK-AD-S Fort Knox, KY 40121-5200
1	Project Manager US Army Tank-Automotive Command M-60 Tank Development ATTN: AMCPM-ABMS (PM Abrams) Warren, MI 48092-2498
1	Commander US Army Training and Doctrine Command ATTN: ATCD-MA, MAJ Williams Fort Monroe, VA 23651
2	Director US Army Materials Technology Laboratory ATTN: SLCMT-ATL Watertown, MA 02172-0001
1	Commander US Army Research Office ATTN: Technical Library P. O. Box 12211 Research Triangle Park, NC 27709-2211

No. of Copies	Organization
1	Commander US Army Belvoir Research and Development Center ATTN: STRBE-WC Fort Belvoir, VA 22060-5006
1	Director US Army TRAC-Ft Lee ATTN: ATRC-L, (Mr. Cameron) Fort Lee, VA 23801-6140
1	Commandant US Army Command and General Staff College Fort Leavenworth, KS 66027
1	Commandant US Army Special Warfare School ATTN: Rev and Tng Lit Div Fort Bragg, NC 28307
3	Commander Radford Army Ammunition Plant ATTN: SMCAR-QA/HI LIB Radford, VA 24141-0298
1	Commander US Army Foreign Science and Technology Center ATTN: AMXST-MC-3 220 Seventh Street, NE Charlottesville, VA 22901-5396
2	Commander Naval Sea Systems Command ATTN: SEA 62R SEA 64 Washington, DC 20362-5101
1	Commander Naval Air Systems Command ATTN: AIR-954-Technical Library Washington, DC 20360
1	Assistant Secretary of the Navy (R, E, and S) ATTN: R. Reichenbach Room 5E787 Pentagon Bldg Washington, DC 20375

No. of Copies	Organization
1	Naval Research Laboratory Technical Library Washington, DC 20375
1	Commandant US Army Command and General Staff College Fort Leavenworth, KS 66027
2	Commandant US Army Field Artillery Center and School ATTN: ATSF-CO-MW, B. Willis Ft. Sill, OK 73503-5600
1	Office of Naval Research ATTN: Code 473, R. S. Miller 800 N. Quincy Street Arlington, VA 22217-9999
3	Commandant US Army Armor School ATTN: ATZK-CD-MS, M. Falkovitch Armor Agency Fort Knox, KY 40121-5215
2	Commander US Naval Surface Warfare Center ATTN: J. P. Consaga C. Gotzmer Indian Head, MD 20640-5000
4	Commander Naval Surface Warfare Center ATTN: Code 240, S. Jacobs Code 730 Code R-13, K. Kim R. Bernecker Silver Spring, MD 20903-5000
2	Commanding Officer Naval Underwater Systems Center ATTN: Code 5B331, R. S. Lazar Technical Library Newport, RI 02840
5	Commander Naval Surface Warfare Center ATTN: Code G33, J. L. East W. Burrell J. Johndrow Code G23, D. McClure Code DX-21 Technical Library Dahlgren, VA 22448-5000

No. of Copies	Organization
3	Commander Naval Weapons Center ATTN: Code 388, C. F. Price Code 3895, T. Parr Information Science Division China Lake, CA 93555-6001
1	Program Manager AFOSR Directorate of Aerospace Sciences ATTN: L. H. Caveny Bolling AFB Washington, DC 20332-0001
5	Commander Naval Ordnance Station ATTN: L. Torreyson T. C. Smith D. Brooks W. Vienna Technical Library Indian Head, MD 20640-5000
1	AL/TSTL (Technical Library) ATTN: J. Lamb Edwards AFB, CA 93523-5000
1	AFSC/SDOA Andrews AFB, MD 20334
1	AFATL/DLYV Eglin AFB, FL 32542-5000
1	AFATL/DLXP Eglin AFB, FL 32542-5000
1	AFATL/DLJE Eglin AFB, FL 32542-5000
1	NASA/Lyndon B. Johnson Space Center ATTN: NHS-22 Library Section Houston, TX 77054
1	AFELM, The Rand Corporation ATTN: Library D 1700 Main Street Santa Monica, CA 90401-3297
3	AAI Corporation ATTN: J. Herbert J. Frankle D. Cleveland P. O. Box 126 Hunt Valley, MD 21030-0126

No. of Copies	Organization
1	Aerojet Ordnance Company ATTN: D. Thatcher 2521 Michelle Drive Tustin, CA 92680-7014
1	Aerojet Solid Propulsion Company ATTN: P. Micheli Sarcramento, CA 96813
1	Atlantic Research Corporation ATTN: M. King 5390 Cherokee Avenue Alexandria, VA 22312-2302
4	AL/LSCF ATTN: J. Levine L. Quinn D. Williams T. Edwards Edwards AFB, CA 93523-5000
1	AVCO Everett Research Laboratory ATTN: D. Stickler 2385 Revere Beach Parkway Everett, MA 02149-5936
2	Calspan Corporation ATTN: C. Murphy P. O. Box 400 Buffalo, NY 14225-0400
1	General Electric Company Armament Systems Department ATTN: M. J. Bulman 128 Lakeside Avenue Burlington, VT 05401-4985
1	IITRI ATTN: M. J. Klein 10 W. 35th Street Chicago, IL 60616-3799
1	Hercules, Inc. Allegheny Ballistics Laboratory ATTN: William B. Walkup P. O. Box 210 Rocket Center, WV 26726
1	Hercules, Inc. Radford Army Ammunition Plant ATTN: J. Pierce Radford, VA 24141-0299

No. of Copies	Organization
3	Lawrence Livermore National Laboratory ATTN: L-355, A. Buckingham M. Finger L-324, M. Constantino P. O. Box 808 Livermore CA 94550-0622
1	Olin Corporation Badger Army Ammunition Plant ATTN: R. J. Thiede Baraboo, WI 53913
1	Olin Corporation Smokeless Powder Operation ATTN: D. C. Mann P. O. Box 222 St. Marks, FL 32355-0222
1	Paul Gough Associates, Inc. ATTN: P. S. Gough P. O. Box 164 1048 South Street Portsmouth, NH 03801-1614
1	Physics International Company ATTN: Library, H. Wayne Wampler 2700 Merced Street San Leandro, CA 98457-5602
1	Princeton Combustion Research Laboratory, Inc. ATTN: M. Summerfield 475 US Highway One Monmouth Junction, NJ 08852-9650
2	Rockwell International Rocketdyne Division ATTN: BA08, J. E. Flanagan J. Gray 6633 Canoga Avenue Canoga Park, CA 91303-2703
3	Thiokol Corporation Huntsville Division ATTN: D. Flanigan Dr. John Deur Technical Library Huntsville, AL 35807

No. of Copies	Organization
2	Thiokol Corporation Elkton Division ATTN: R. Biddle Technical Library P. O. Box 241 Elkton, MD 21921-0241
1	Veritay Technology, Inc. ATTN: E. Fisher 4845 Millersport Highway East Amherst, NY 14501-0305
1	Universal Propulsion Company ATTN: H. J. McSpadden Black Canyon Stage 1 Box 1140 Phoenix, AZ 84029
1	Battelle Memorial Institute ATTN: Technical Library 505 King Avenue Columbus, OH 43201-2693
1	Brigham Young University Department of Chemical Engineering ATTN: M. Beckstead Provo, UT 84601
1	California Institute of Technology 204 Karman Laboratory Main Stop 301-46 ATTN: F.E.C. Culick 1201 E. California Street Pasadena, CA 91109
1	California Institute of Technology Jet Propulsion Laboratory ATTN: L. D. Strand, MS 512/102 4800 Oak Grove Drive Pasadena, CA 91109-8099
1	University of Illinois Department of Mechanical/Industrial Engineering ATTN: H. Krier 144 MEB; 1206 N. Green Street Urbana, IL 61801-2978
1	University of Massachusetts Department of Mechanical Engineering ATTN: K. Jakus Amherst, MA 01002-0014

No. of Copies	Organization
1	University of Minnesota Department of Mechanical Engineering ATTN: E. Fletcher Minneapolis, MN 55414-3368
1	Case Western Reserve University Division of Aerospace Sciences ATTN: J. Tien Cleveland, OH 44135
3	Georgia Institute of Technology School of Aerospace Engineering ATTN: B.T. Zim E. Price W.C. Strahle Atlanta, GA 30332
1	Institute of Gas Technology ATTN: D. Gidaspow 3424 S. State Street Chicago, IL 60616-3896
1	Johns Hopkins University Applied Physics Laboratory Chemical Propulsion Information Agency ATTN: T. Christian Johns Hopkins Road Laurel, MD 20707-0690
1	Massachusetts Institute of Technology Department of Mechanical Engineering ATTN: T. Toong 77 Massachusetts Avenue Cambridge, MA 02139-4307
1	Pennsylvania State University Applied Research Laboratory ATTN: G.M. Faeth University Park, PA 16802-7501
1	Pennsylvania State University Department of Mechanical Engineering ATTN: K. Kuo University Park, PA 16802-7501
1	Purdue University School of Mechanical Engineering ATTN: J. R. Osborn TSPC Chaffee Hall West Lafayette, IN 47907-1199

No. of Copies	Organization
1	SRI International Propulsion Sciences Division ATTN: Technical Library 333 Ravenwood Avenue Menlo Park, CA 94025-3493
1	Rensselaer Ploytechnic Institute Department of Mathematics Troy, NY 12181
2	Director Los Alamos Scientific Laboratory ATTN: T3, D. Butler M. Division, B. Craig P. O. Box 1663 Los Alamos, NM 87544
1	General Applied Sciences Laboratory ATTN: J. Erdos 77 Raynor Avenue Ronkonkama, NY 11779-6649
1	Battelle PNL ATTN: Mr. Mark Garnich P. O. Box 999 Richland, WA 99352
1	Stevens Institute of Technology Davidson Laboratory ATTN: R. McAlevy, III Castle Point Station Hoboken, NJ 07030-5907
1	Rutgers University Department of Mechanical and Aerospace Engineering ATTN: S. Temkin University Heights Campus New Brunswick, NJ 08903
1	University of Southern California Mechanical Engineering Department ATTN: 0HE200, M. Gerstein Los Angeles, CA 90089-5199
2	University of Utah Department of Chemical Engineering ATTN: A. Baer G. Flandro Salt Lake City, UT 84112-1194

No. of Copies	Organization
1	Washington State University Department of Mechanical Engineering ATTN: C. T. Crowe Pullman, WA 99163-5201
1	Honeywell, Inc. ATTN: R. E. Tompkins MN38-3300 10400 Yellow Circle Drive Minnetonka, MN 55343
1	Science Applications, Inc. ATTN: R. B. Edelman 23146 Cumorah Crest Drive Woodland Hills, CA 91364-3710
	<u>Aberdeen Proving Ground</u>
	Cdr, CSTA ATTN: STECS-LI, R. Hendricksen

INTENTIONALLY LEFT BLANK.